AQA A-Level Further Maths 2025 Paper 1

Do nut turn over the page until instructed to do so.

This assessment is out of 100 marks and you will be given 120 minutes.

When you are asked to by your teacher write your **full name** below

Name:

Total Marks: / 100

1 The function $y = \tanh(x)$ can be defined to be:

$$\frac{e^{x} + e^{-x}}{2} \qquad \frac{e^{x} + e^{-x}}{e^{x} - e^{-x}} \qquad \frac{e^{2x} - 1}{e^{2x} + 1} \qquad \frac{e^{2x} + 1}{e^{2x} - 1}$$

[1 mark]

2 The vector $\mathbf{v}$ is an eigenvector of the matrix $\mathbf{A}$ with corresponding eigenvalue 3.

Given that $\mathbf{A}^2\mathbf{v} = \lambda\mathbf{v}$ what is the value of λ?

$$3 \qquad 9 \qquad \frac{1}{3} \qquad \frac{1}{9}$$

[1 mark]

3 The gradient of $y = \arctan(x)$ at $x = \dfrac{1}{3}$ is

$\dfrac{9}{10}$ $\dfrac{3\sqrt{2}}{4}$ $\dfrac{9}{8}$ $\dfrac{10}{9}$

[1 mark]

4 State the vertices and asymptotes of the hyperbola

$$\frac{x^2}{9} - 4y^2 = 1$$

[2 marks]

5 The plane Π contains the vectors $\mathbf{u} = \mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $\mathbf{v} = 3\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ and also the point $(3,0,2)$.

Find the Cartesian equation of the plane.

[4 marks]

6 The matrix T represents a stretch scale factor 3 parallel to the $x-$axis followed by a rotation $90°$ anticlockwise centre the origin.
T is applied to the point $P(a, b)$ resulting in the image $P'(-3,6)$.

Find the values of a and b.

[4 marks]

7

a) Express in the form $\dfrac{A}{r+2} + \dfrac{B}{r+3}$ the function

$$f(r) = \frac{1}{r^2 + 5r + 6}$$

[2 marks]

b) Hence show that $\displaystyle\sum_{r=1}^{n} \frac{2}{r^2 + 5r + 6} = \frac{an}{bn + c}$ where a, b and c are integers to be determined.

[5 marks]

8 The region R is enclosed by the curve $y = \sqrt[3]{x}$ and the lines $y = \frac{1}{2}x - 2$, $x = 8$ and $y = 0$.

Find, in the form $m\pi$, $m \in \mathbb{Q}$, the volume of the solid formed when the region R is rotated $360°$ around the $x-$axis.

[6 marks]

9 Find the value of $\lim\limits_{x \to 0} \left(\cos(x)\right)^{\frac{1}{x}}$

[5 marks]

10 For $z \in \mathbb{C}$, solve the equation $3z^2 + 2iz^* = -38 + 96i$

[8 marks]

11 Find the general solution for the differential equation

$$x\frac{\mathrm{d}y}{\mathrm{d}x} + 2y = \frac{1}{x\sqrt{1+x^2}}$$

[7 marks]

12

a) Starting with the identity $\cosh^2(x) - \sinh^2(x) = 1$ derive an \ identity involving $\coth^2(x)$

[2 marks]

b) Find, in exact form, the solutions of

$$\operatorname{cosech}^2(x) + \coth^2(x) - \operatorname{cosech}(x) = 7$$

[6 marks]

13

a) Sketch the polar curve $r = \sin(2\theta)$, $r > 0$

[3 marks]

b) Find the area enclosed by the curve, C, showing all reasoning.

[5 marks]

14) **a)** Find the coordinates of the stationary points of the function

$$f(x) = \frac{x(x+6)}{x+8}$$

[7 marks]

b) Find the equation of the oblique asymptote of $f(x)$.

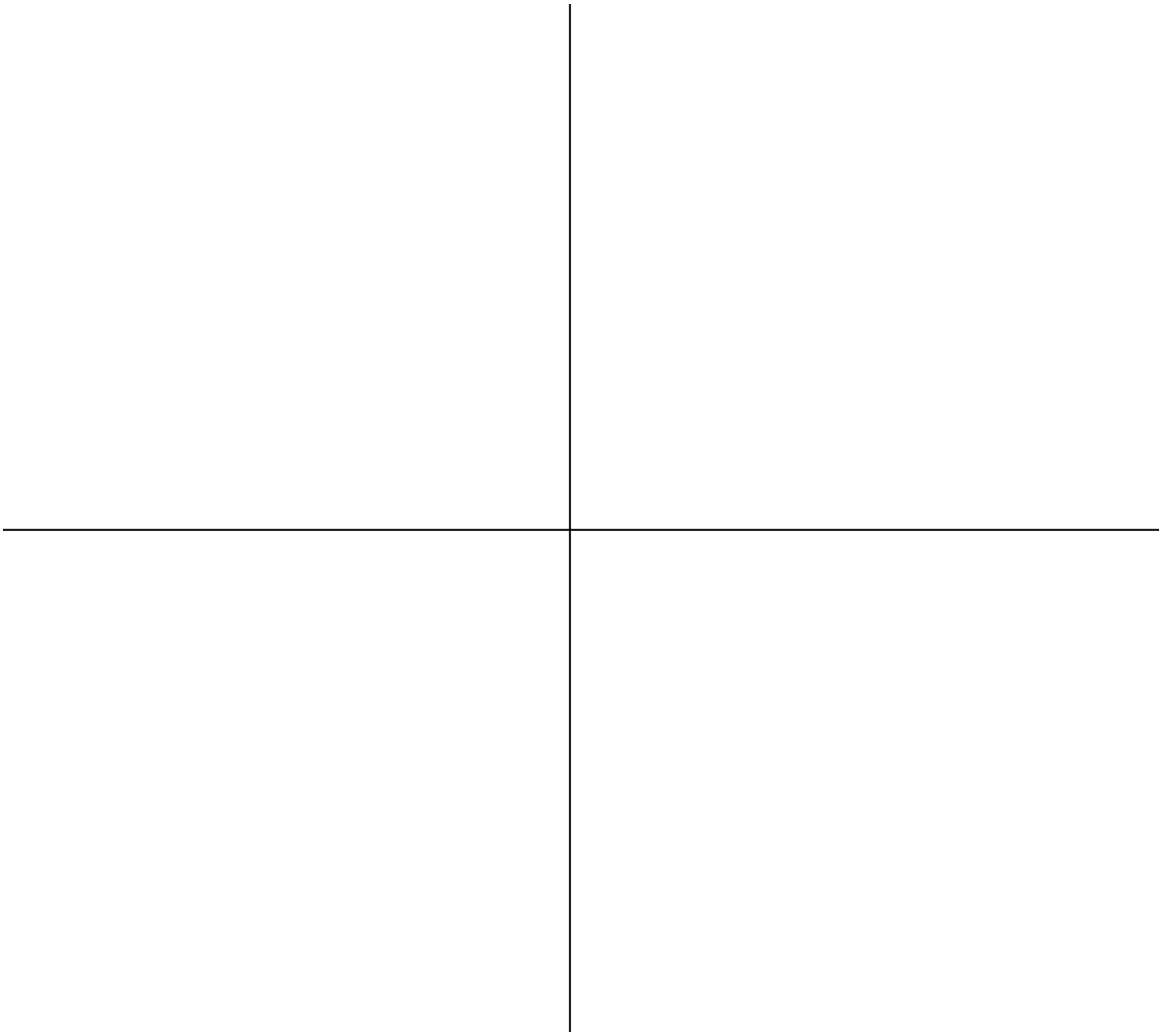
[2 marks]

c) State the interval of y for which $f(x)$ doesn't take a value.

[1 mark]

d) Hence, sketch the graph of $y = f(x)$ on the axes below.

[3 marks]



15 Let,

$$S = \mathrm{e}^{\mathrm{i}\theta} + \mathrm{e}^{2\mathrm{i}\theta} + \mathrm{e}^{3\mathrm{i}\theta} + \ldots + \mathrm{e}^{6\mathrm{i}\theta}$$

for $\theta \neq 2n\pi, \quad n \in \mathbb{Z}$.

Show that

$$S = \frac{\mathrm{e}^{\frac{7\mathrm{i}\theta}{2}} \sin\left(3\theta\right)}{\sin\left(\frac{\theta}{2}\right)}$$

and find an expression for

$$W = \sin(\theta) + \sin(2\theta) + \sin(3\theta) + \sin(4\theta) + \sin(4\theta) + \sin(5\theta) + \sin(6\theta)$$

[8 marks]

16

a) The line l_1 passes through the points $A(1,3,2)$ and $B(5,5,4)$.

Find the shortest distance between $P(3,2,-7)$ and the line.

[5 marks]

b) State the coordinates of the point Q, lying on l_1, at which the line perpendicular to l_1 and passing through P intersects l_1.

[1 mark]

c) Hence find points R on l_2 such that $|RP| = 2|PQ|$

[4 marks]

17 Prove the following result by induction

$$\frac{\mathrm{d}^n}{\mathrm{d}x^n}\left[\cos(ax)\right] = a^n \cos\left(ax + \frac{n\pi}{2}\right)$$

where $\dfrac{\mathrm{d}^n}{\mathrm{d}x^n}$ denotes the nth derivative with respect to x.

[8 marks]