

## **AQA A-Level Further Maths 2026 Paper 1**

Do not turn over the page until instructed to do so.

This assessment is out of 100 marks and you will be given 120 minutes.

When you are asked to by your teacher write your **full name** below

**Name:**

**Total Marks:**                      **/ 100**

- 1 The complex number  $z = 2 - 5i$  is a root of the quadratic equation  $az^2 + bz + c$  where  $a, b, c \in \mathbb{R}$ ,

What is the other root?

$$-2 - 5i$$

$$2 + 5i$$

$$-2 + 5i$$

$$4 - 10i$$

[1 mark]

- 2 Let  $\lambda = 4$  be an eigenvalue of the matrix  $\mathbf{A}$  with associated eigenvector  $\mathbf{v}$ .

Which of the below must be an eigenvalue of the matrix  $\mathbf{A}^2$

$$4$$

$$2$$

$$\frac{1}{4}$$

$$16$$

[1 mark]

- 3 A particle undergoes simple harmonic motion according to the equation

$$\ddot{x} = -16x$$

where  $x$  metres is the displacement from the central line at time  $t$  seconds.

Given that the maximum displacement is 4 m what is the maximum speed of the particle?

16 ms<sup>-1</sup>

8 ms<sup>-1</sup>

64 ms<sup>-1</sup>

256 ms<sup>-1</sup>

**[1 mark]**

- 4 Given that  $\begin{pmatrix} 3 \\ a \\ 1 \end{pmatrix} \times \begin{pmatrix} 4 \\ 0 \\ b \end{pmatrix} = \begin{pmatrix} 10 \\ -11 \\ -8 \end{pmatrix}$  then which of the following is true?

$$a = 2, b = 5$$

$$a = 2, b = -5$$

$$a = -2, b = 5$$

$$a = -2, b = 5$$

**[1 mark]**

5 a) Express  $\frac{2}{(r+1)(r+3)}$  in the form  $\frac{A}{r+1} + \frac{B}{r+3}$

**[2 marks]**

b) Hence, find  $\sum_{r=1}^n \frac{2}{(r+1)(r+3)}$

**[4 marks]**



c) Find,

$$\frac{1}{12} + \frac{2}{45} + \frac{2}{36} + \dots + \frac{2}{380}$$

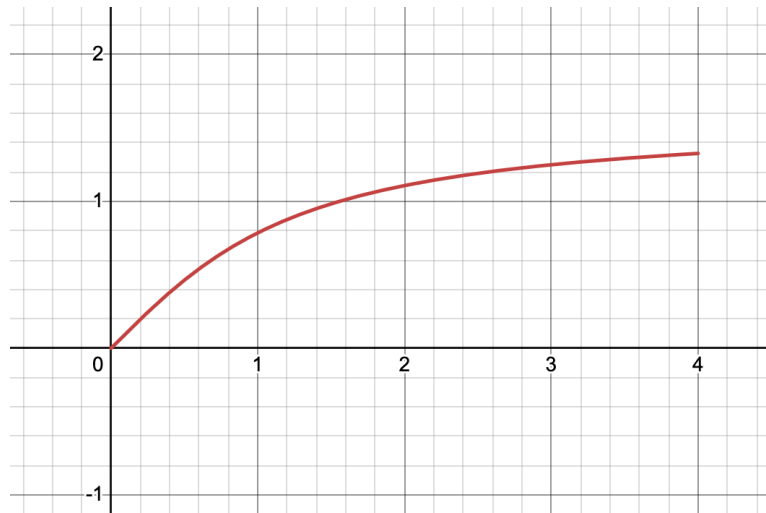
**[2 marks]**

- 6** Find the angle between the line which passes through the points  $A(1,2,1)$  and  $B(4,5,2)$  and the plane with equation  $3x + 4y - z = 2$ .

**[5 marks]**

7

The graph below shows  $y = \arctan(x)$  for  $0 \leq x \leq 4$ .



The region enclosed by  $y = \arctan(x)$ , and the lines  $x = 0$ ,  $x = 4$  and the  $x$ -axis is rotated by  $2\pi$  radians about the  $x$ -axis.

Using Simpson's rule with 5 ordinates, estimate the volume of the solid formed.

Give your answer to 4 decimal places.

**[5 marks]**



8 Solve the hyperbolic equation

$$7 - 2 \cosh^2(x) = \sinh^2(x) - 5 \sinh(x) + 3$$

**[5 marks]**

**9 a)** Given that  $z = \cos(\theta) + i \sin(\theta)$ ,

**(i)** Prove that  $z + \frac{1}{z} = 2 \cos(\theta)$

**[2 marks]**

**(ii)** Derive a similar result for  $z^n + \frac{1}{z^n}$

**[2 marks]**

**b)** Using the results established in (a), show that

$$\int_0^{\frac{\pi}{3}} \cos^5(\theta) \, d\theta = \frac{a\sqrt{3}}{b}$$

where  $a$  and  $b$  are integers to be found.

**[7 marks]**



**10 a)** Let  $y = \arccos(x)$ ,  $-1 \leq x \leq 1$  show that

$$\frac{d}{dx} (\arccos(x)) = -\frac{1}{\sqrt{1-x^2}}$$

**[3 marks]**

**b)** Hence, show that

$$\int_0^{2\sqrt{2}} \frac{3x - 2}{\sqrt{16 - x^2}} dx = 2a - a\sqrt{2} + \frac{\pi}{a}$$

where  $a$  is an integer to be found.

**[7 marks]**

11 a) Let  $\mathbf{A} = \begin{pmatrix} 2k & -2 \\ 3 + 2k & k \end{pmatrix}$

Show that the matrix  $\mathbf{A}$  is non-singular for all values of  $k$ .

**[2 marks]**

b)  $\mathbf{A}$  represents the transformation  $T : \mathbb{R} \mapsto \mathbb{R}^3$ .

Given that the point  $(a, 3a)$  is mapped to  $(-6, 39)$ , find  $k$  and  $a$ .

**[5 marks]**



**12** Consider  $f(x) = \frac{x^2 + 3x}{(3x + 1)(x + 1)}$

**a)** What are the asymptotes of  $f(x)$ .

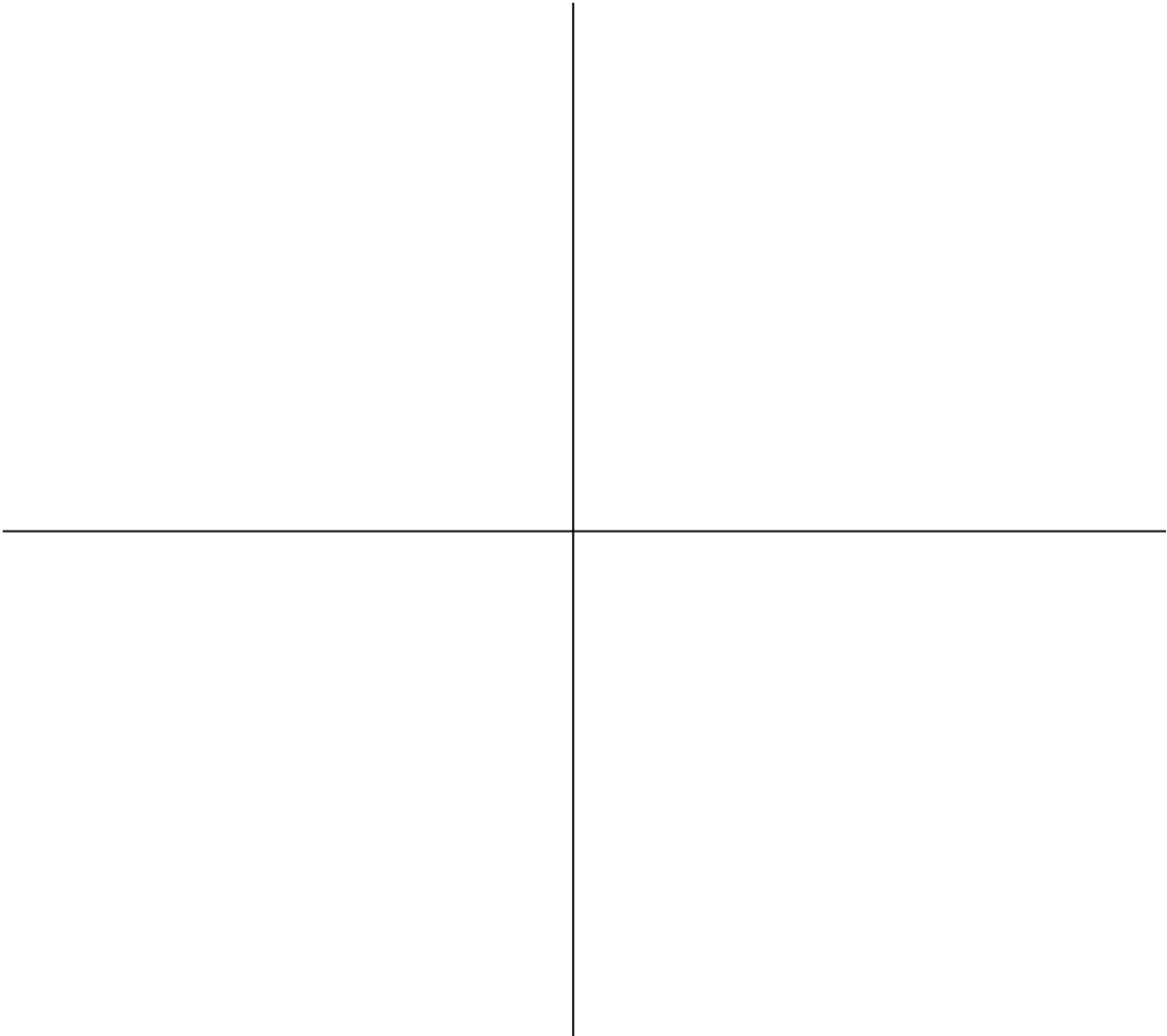
**[3 marks]**

**b)** Find, fully justifying your working and without using calculus, the range of  $f(x)$ .

**[5 marks]**



- c) Sketch, on the axes below, the graph.



**[3 marks]**

- d) What are the coordinates of the maximum value of  $\frac{1}{f(x)}$  ?

**[1 mark]**

**13**

The curve  $C$  has polar equation

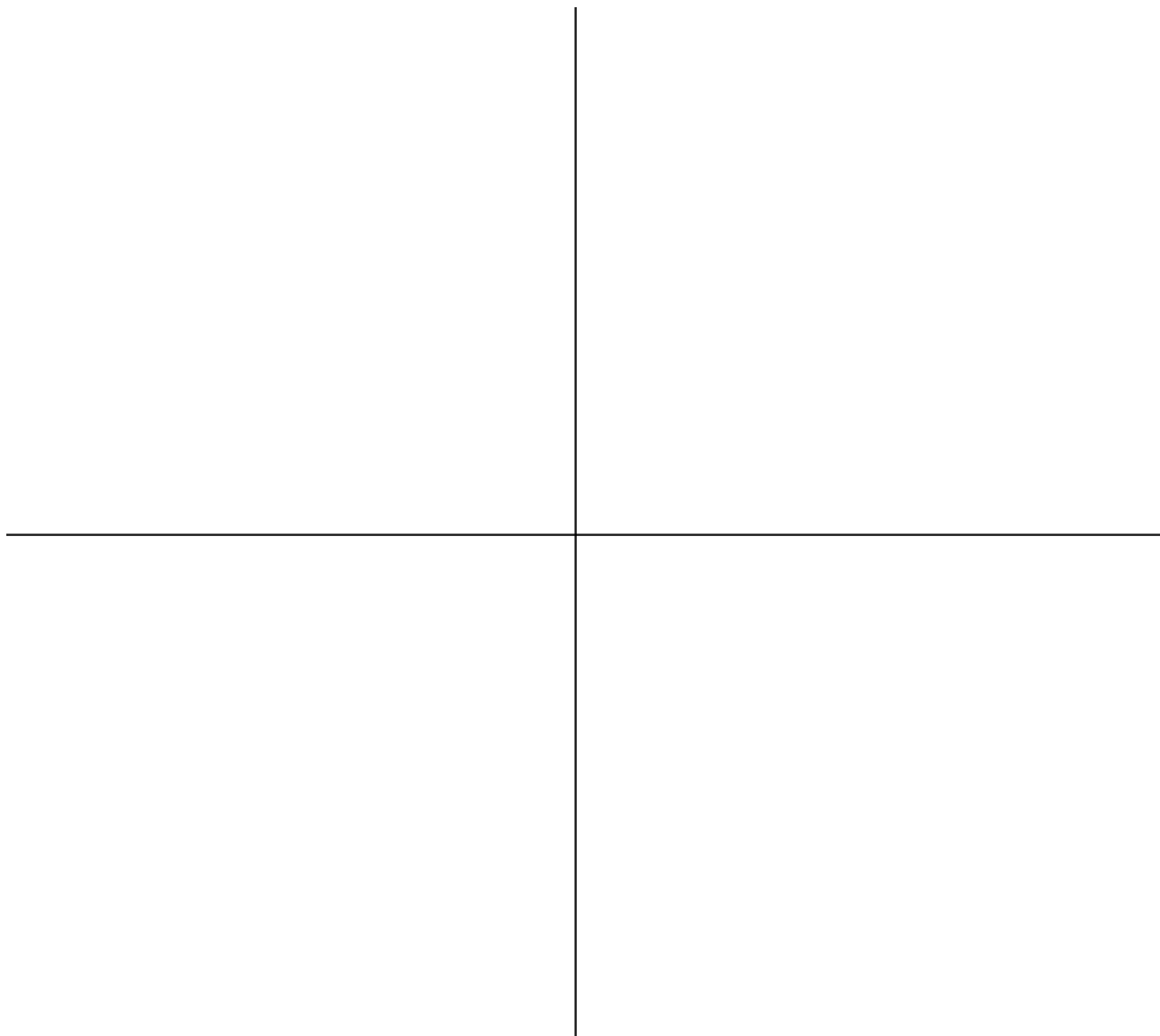
$$r = \frac{3}{\sqrt{4 \cos^2(\theta) + 16 \sin^2(\theta)}}$$

- a)** Find the Cartesian equation of the curve.

**[4 marks]**

b) Sketch the graph of  $C$

**[2 marks]**



14 Given that  $y = 5$  when  $x = \ln(3)$ , Solve the differential equation

$$\frac{dy}{dx} + y \coth(x) = \cosh^2(x)$$

**[7 marks]**



- 15) The roots of the polynomial  $p(x) = 2x^3 - 17x^2 + 41x - 30$  are  $\alpha$ ,  $\beta$  and  $\gamma$ .

Find the polynomial with roots  $\alpha^2 + \alpha$ ,  $\beta^2 + \beta$  and  $\gamma^2 + \gamma$ .

**[9 marks]**





**16** Two tanks,  $A$  and  $B$ , each of capacity 500 litres.

At time  $t = 0$  both tanks are full of pure water.

A tap is turned on and the following flows of water occur.

- Salt water of concentration 0.5 grams per litre flows into Tank  $A$  at 30 litres per minute.
- Water flows from Tank  $A$  to Tank  $B$  at 20 litres per minute.
- Water flows from Tank  $B$  to Tank  $A$  at 5 litres per minute.
- Water flows out of Tank  $B$  through a waste pipe.
- The amount of water remains at 500 litres in each tank.

At time  $t$ ,  $x$  is the number of grams of salt in Tank  $A$  and  $y$  is the number of grams of salt in  $B$ .

- a)** Show that the following system of coupled differential equations holds.

$$\frac{dx}{dt} = 15 - 0.04x + 0.01y$$
$$\frac{dy}{dt} = 0.04x - 0.04y$$

**[3 marks]**

**b)** Solve the differential equations giving  $x$  and  $y$  as functions of  $t$ .



