

AQA A-Level Further Maths 2026 Paper 2

Do not turn over the page until instructed to do so.

This assessment is out of 100 marks and you will be given 120 minutes.

When you are asked to by your teacher write your **full name** below

Name:

Total Marks: **/ 100**

- 1 The matrix \mathbf{M} represents a rotation about the y -axis.

$$\mathbf{M} = \begin{pmatrix} \frac{\sqrt{3}}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \\ a & 0 & b \end{pmatrix}$$

Which of the following pairs is correct?

- a) $a = -\frac{1}{2}$. $b = -\frac{\sqrt{3}}{2}$
- b) $a = -\frac{1}{2}$. $b = \frac{\sqrt{3}}{2}$
- c) $a = \frac{1}{2}$. $b = -\frac{\sqrt{3}}{2}$
- d) $a = \frac{1}{2}$. $b = \frac{\sqrt{3}}{2}$

[1 mark]

- 2 Given that $y = \operatorname{cosech}(x)$. Find $\frac{dy}{dx}$

- a) $\operatorname{sech}(x)\tanh(x)$
- b) $-\operatorname{sech}(x)\tanh(x)$
- c) $\operatorname{cosech}(x)\coth(x)$
- d) $-\operatorname{cosech}(x)\coth(x)$

[1 mark]

- 3** What is the geometric configuration for the system of equations below?

$$\begin{cases} x + 3y - 7z & = 8 \\ 2x + 0y - 2z & = -2 \\ x - y + z & = -4 \end{cases}$$

- a)** Unique solution (planes intersections at a single point)
- b)** Sheaf
- c)** Triangular prism
- d)** The two planes are parallel

[1 mark]

4 Show, fully justifying your answer, that

$$\int_{-\infty}^{\infty} xe^{-x^2} dx = 0$$

[5 marks]

5 Let \mathbf{M} be the matrix $\mathbf{M} = \begin{pmatrix} 2 & 0 \\ -\frac{1}{2} & 3 \end{pmatrix}$

a) Find the eigenvalue-eigenvector pairs for the matrix \mathbf{M}

[4 marks]

- b) Without calculating \mathbf{M}^{-1} state the eigenvalues and eigenvectors of \mathbf{M}^{-1} .

[2 marks]

- 6** The point $A(3,2,1)$ lies on a line which is perpendicular to the vectors $\mathbf{u} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$ and $\mathbf{v} = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$.
- a)** Find the Cartesian equation of the line.

[5 marks]

- b)** Show that the point $P(15, -13, -20)$ lies on the line and find the distance $|AP|$.

[3 marks]

7 Let $I_n = \int \tanh^n(x) \, dx$.

a) Prove, that for $n \geq 2$,

$$I_n = I_{n-2} - \frac{1}{n-1} \tanh^{n-1}(x)$$

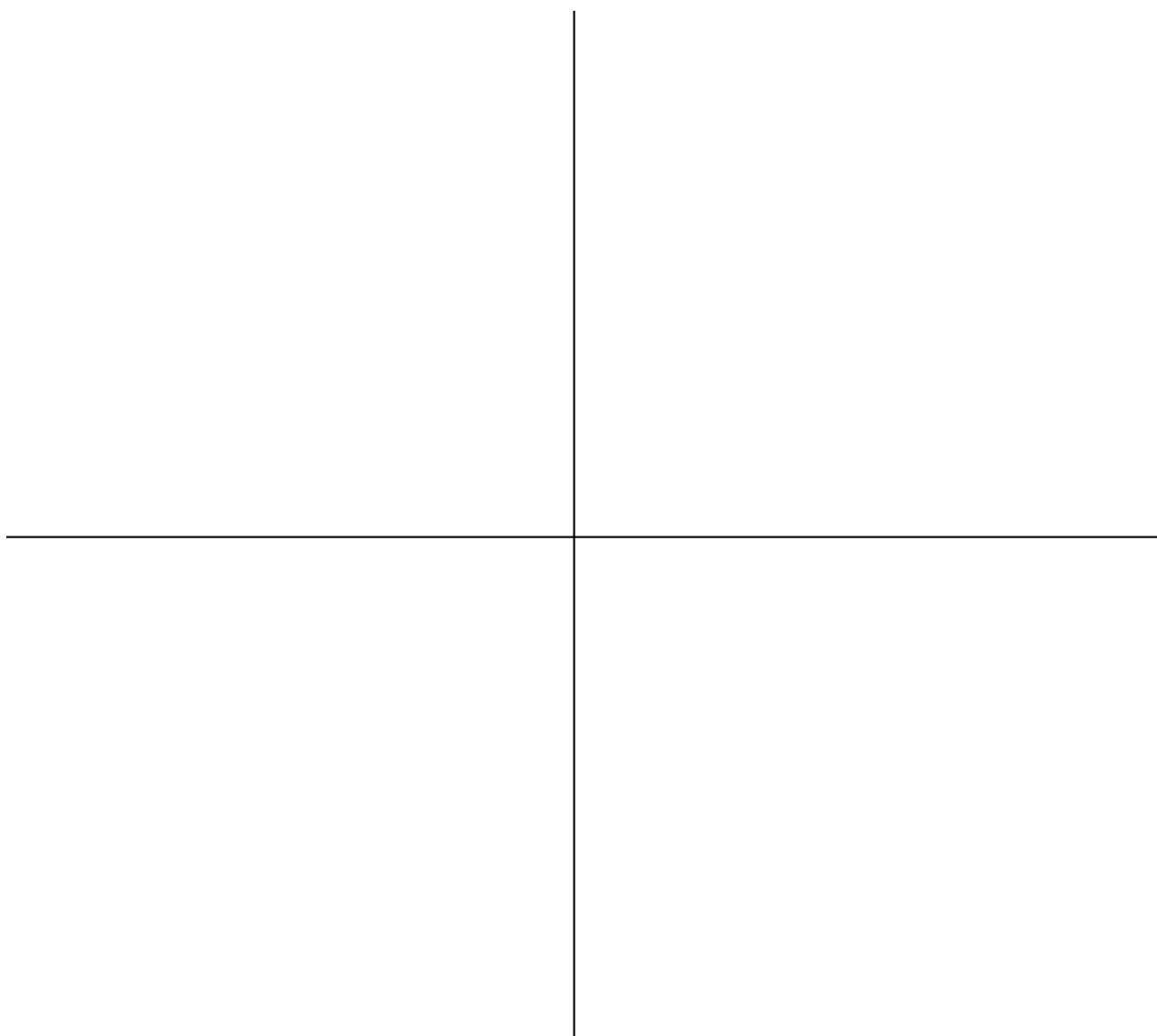
[4 marks]

b) Hence, find I_6

[3 marks]

8

- a)** Sketch on the axes below $y = \operatorname{artanh}(2x)$ and state its domain and range.

**[4 marks]**

b) Derive the logarithmic form of $y = \operatorname{artanh}(2x)$

[4 marks]

- c) Solve, giving your answer in an exact form,

$$6 + \tanh(x) = 12\operatorname{sech}^2(x)$$

[4 marks]

9

The function f is defined by

$$f(x) = \frac{x^2 - 4x + 5}{x - 3}, \quad x \in \mathbb{R}, x \neq 3$$

- a)** Find the interval (a, b) in which $f(x)$ does not take a value.

Fully justify your answer.

[5 marks]

- b)** Find the coordinates of the two stationary points of the graph $y = f(x)$.

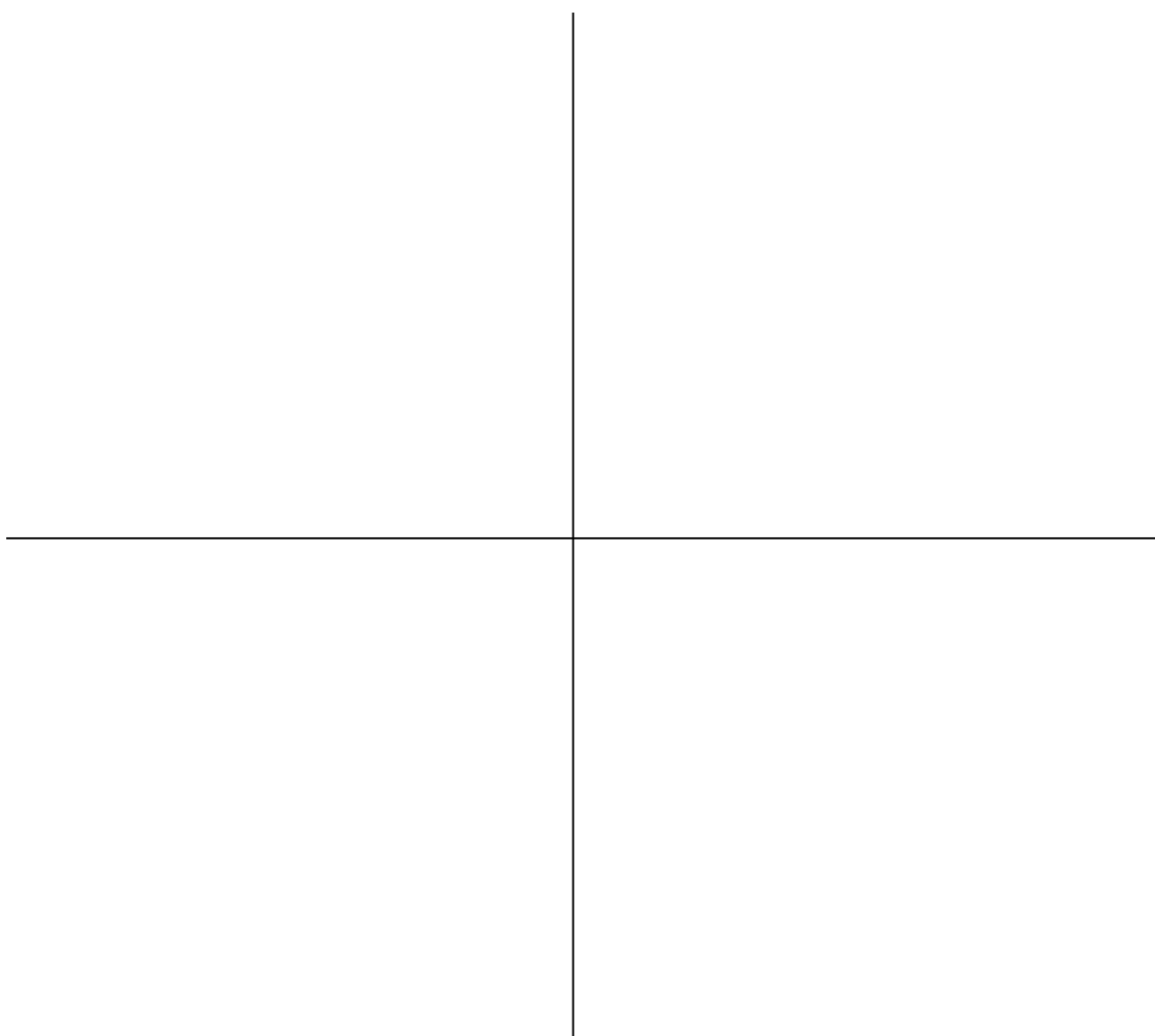
[3 marks]

- c)** Show that the curve has an oblique asymptote and find its equation.

[2 marks]

d) Sketch the graph of $y = f(x)$ on the axes below.

[4 marks]



- 10 a) Solve, given that at $x = 0$, $y = 2$, the differential equation

$$\frac{dy}{dx} - y \tan(x) = 2 \sin(x)$$

[6 marks]

- b)** George uses Euler's method with a stepwise of $h = 0.1$ to find an estimate of the value of y when $x = 0.3$ for the differential equation defined in (a).

Assuming he performs the calculations correctly, if he begins with $x_0 = 1.1$ and $y_0 = 2.0075$, what does George calculate $y(0.3)$ to be.

[7 marks]

11 Consider the following inhomogeneous differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 3y = e^{-3x} + \cos(x)$$

a) Find the complementary function for this differential equation. **[3 marks]**

b) Lucy decides to try $y = \lambda e^{-3x} + \mu \cos(x)$ as a particular integral.

Give two reasons why this is not a suitable choice of trial function.

[2 marks]

- c) Given that at $x = 0$, $y = \frac{14}{5}$ and $\frac{dy}{dx} = \frac{17}{20}$ solve this differential equation.

[6 marks]

12**a)** Use De Moivre's theorem to show that

$$\tan(6\theta) = \frac{6 \tan(\theta) - 20 \tan^3(\theta) + 6 \tan^5(\theta)}{1 - 15 \tan^2(\theta) + 15 \tan^4(\theta) - \tan^6(\theta)}$$

[6 marks]

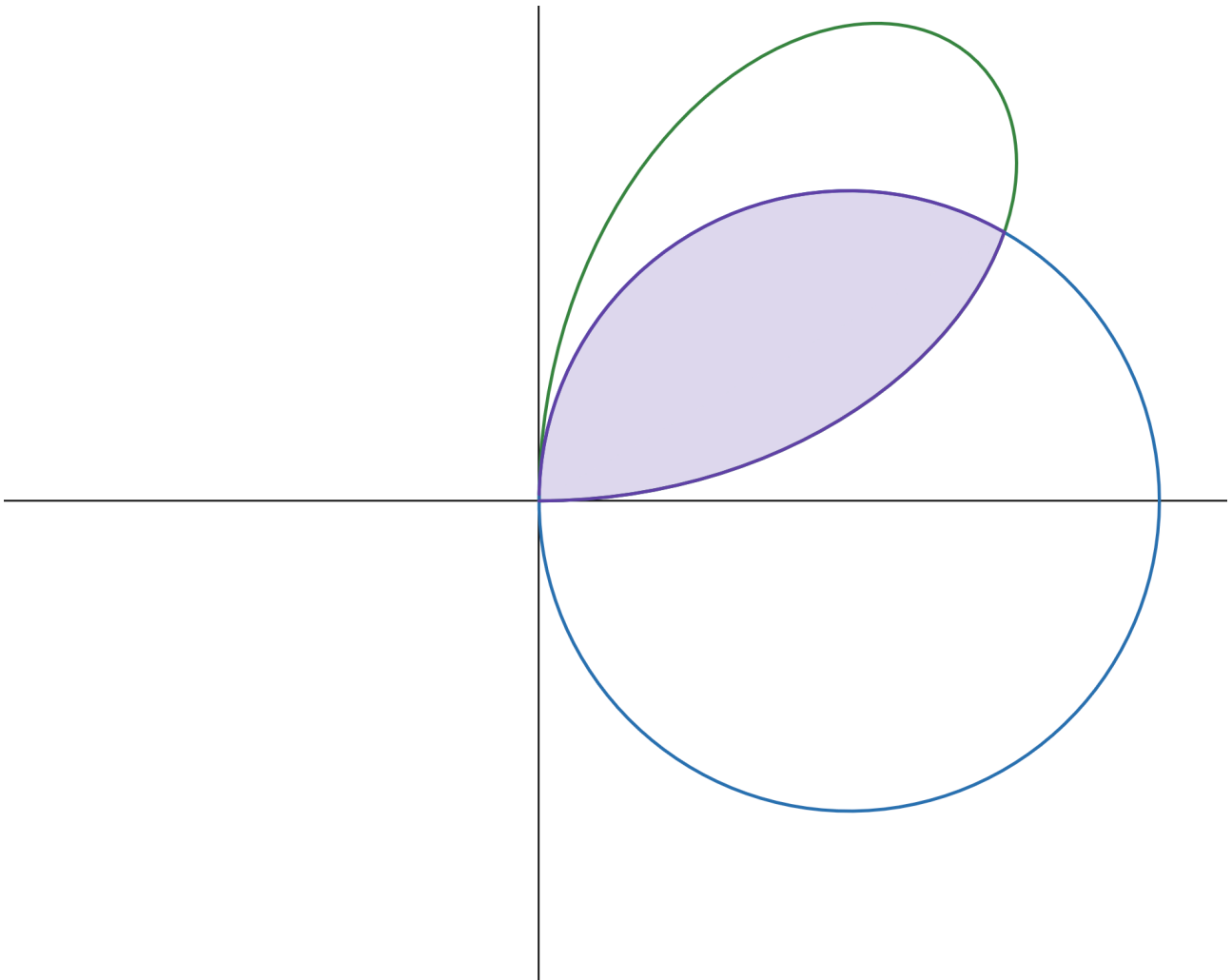
- b)** Hence, show that $t = \tan\left(\frac{\pi}{24}\right)$ is a solution of the polynomial equation $t^4 + 8t^3 + 2t^2 - 8t + 1 = 0$

[4 marks]

13

The diagram below shows the polar curve C_1 with equation $r = 3 \cos(\theta)$, $0 \leq \theta \leq \pi$.

It also shows part of the polar curve C_2 with equation $r = 3 \sin(2\theta)$, $0 \leq \theta \leq 2\pi$.



a) On the diagram above, complete the sketch of C_2

[2 marks]

b) Show that the shaded area is $\frac{9\pi}{8} - \frac{27\sqrt{3}}{32}$.

[9 marks]

