Dr Bennison
Maths
MATHEMATICS EDUCATION EXPERT

AQA A-Level Maths 2026 Paper 1

Do n0t turn over the page until instructed to do so.

This assessment is out of 100 marks and you will be given 120 minutes.

When you are asked to by your teacher write your **full name** below

Name:

Total Marks: / 100

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1 Write as a single logarithm

$\log_{10}(5x) + 2\log_{10}(x)$

$\log_{10}(5x^3)$ $\log_{10}(5x^2)$ $\log_{10}(5)$ $\frac{5}{2}\log_{10}(x)$

[1 mark]

$\log_{10}(5x) + 2\log_{10}(x) = \log_{10}(5x) + \log_{10}(x^2)$

$= \log_{10}(5x \times x^2)$

$= \log_{10}(5x^3)$

2 Which of the following binomial expressions isn't valid for $x = \frac{1}{3}$.

$(1+2x)^{-1}$

$|2x| < 1 \Rightarrow |x| < \frac{1}{2}$

$(1+5x)^{-\frac{1}{2}}$ $|5x| < 1 \Rightarrow |x| < \frac{1}{5}$

$(2+3x)^{-\frac{1}{3}}$

$= 2^{-\frac{1}{3}}\left(1+\frac{3}{2}x\right)^{-\frac{1}{3}}$

$\Rightarrow \left|\frac{3x}{2}\right| < 1$

$\Rightarrow |x| < \frac{3}{3}$

$(3+5x)^{-2}$

$= 3^{-2}\left(1+\frac{5}{3}x\right)^{-2}$

[1 mark]

$\Rightarrow \left|\frac{5}{3}x\right| < 1$

$\Rightarrow |x| < \frac{3}{5}$

3 It is given that

$$\int_0^5 f(x)\,\mathrm{d}x = 15 \qquad \text{and} \qquad \int_0^2 f(x)\,\mathrm{d}x = -5$$

Find the value of $\int_2^5 f(x)\,\mathrm{d}x$.

10 −10 −20 20

[1 mark]

$20 + -5 = 15$

$\int_2^5 f(x)\,dx = 20$

4 A curve with equation $y = f(x)$ passes through the point $(4,9)$.

Given that $f'(4) = 0$ find the equation of the tangent to the curve at $(4,9)$.

Circle your answer.

$y = 9$ $\quad x = 4$ $\quad y = \frac{9}{4}$ $\quad y = 0$

[1 mark]

5 A function $f(x)$ is such that $f(4) = 6$ and $f'(x) = \dfrac{2x^3 + \sqrt{x}}{x}$.

Find an expression for $f(x)$.

[4 marks]

$f'(x) = \dfrac{2x^3 + x^{1/2}}{x}$

$= 2x^2 + x^{-1/2}$

$\Rightarrow f(x) = \int 2x^2 + x^{-1/2}\, dx$

$= \frac{2}{3}x^3 + \frac{1}{1/2}x^{1/2} + C$

$= \frac{2}{3}x^3 + 2\sqrt{x} + C$

When $x = 4$, $f(4) = 6$, so

$6 = \frac{2}{3} \times 4^3 + 2\times\sqrt{4} + C$

$\Rightarrow C = 6 - \frac{140}{3} = \frac{-122}{3}$

So $f(x) = \frac{2}{3}x^3 + 2\sqrt{x} - \frac{122}{3}$

6 Let $p(x)$ be a polynomial such that

$$p(x) = 3x^3 + ax^2 + bx - 24$$

Where a and b are real valued constants.

Given that

- $(x - 3)$ is a factor of $p(x)$,
- and $p'(2) = 22$,

find the values of the constants a and b and fully factorise $p(x)$.

[5 marks]

As $(x-3)$ is a factor, by the factor theorem $p(3) = 0$. So,

$$3\times 3^3 + a\times 3^2 + b\times 3 - 24 = 0$$

$$81 + 9a + 3b - 24 = 0$$

$$9a + 3b = -57 \quad (1)$$

$\Rightarrow$

$$p'(x) = 9x^2 + 2ax + b$$

So $p'(2) = 22 \Rightarrow 9\times 4 + 4a + b = 22$

$$\Rightarrow 4a + b = -14 \quad (2)$$

Solving (1) and (2) we obtain $a = 5$, $b = -34$

Hence,

$$p(x) = 3x^3 + 5x^2 - 34x - 24$$
$$= (x-3)(3x^2 + 14x + 8)$$
$$= (x-3)(3x+2)(x+4)$$

7 Let $a, b \in \mathbb{R}$. Prove that $\frac{a+b}{2} \geq \sqrt{ab}$

[4 marks]

Assume for a contradiction that

$$\frac{a+b}{2} < \sqrt{ab}$$

Then,

$$\left(\frac{a+b}{2}\right)^2 < ab$$

$$\Rightarrow \frac{1}{4}(a+b)^2 < ab$$

$$\Rightarrow (a+b)^2 < 4ab$$

$$\Rightarrow a^2+2ab+b^2 < 4ab$$

$$\Rightarrow a^2-2ab+b^2 < 0$$

$$\Rightarrow (a-b)^2 < 0$$

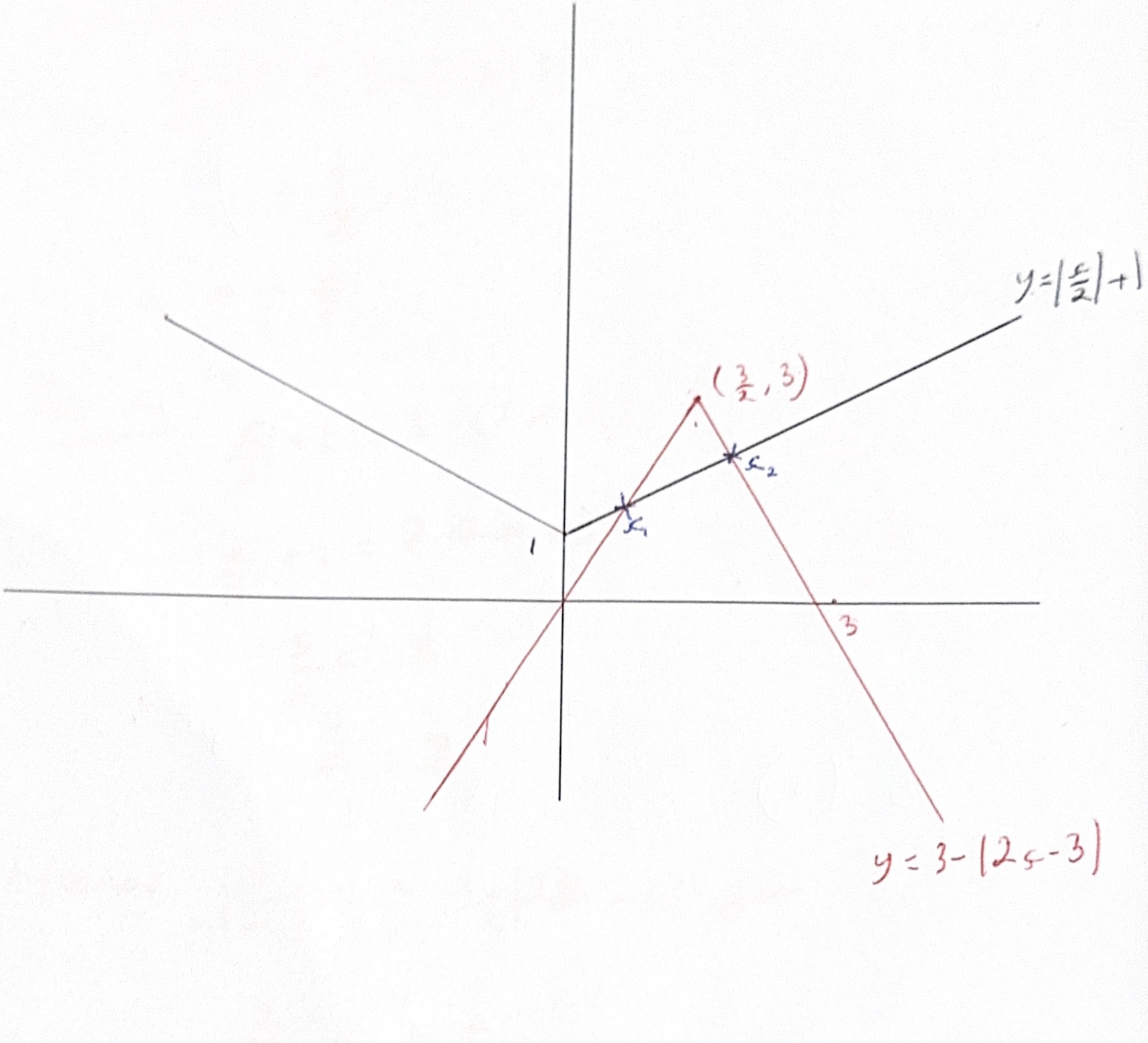
which is a contradiction since $(\cdot)^2 \geq 0$ always.

Hence, by contradiction,

$$\frac{a+b}{2} \geq \sqrt{ab}$$

8 a) Sketch on the axes below the graphs of $y = 3 - |2x - 3|$ and $y = \left|\frac{x}{2}\right| + 1$.

[3 marks]



b) Hence, solve the inequality

$$\left|\frac{x}{2}\right| + 1 \leq 3 - |2x - 3|$$

[4 marks]

For x_1:

$$\frac{x}{2} + 1 = 3 + (2x - 3)$$

$$1 = \frac{3}{2}x$$

$$x = \frac{2}{3}$$

For x_2:

$$\frac{x}{2} + 1 = 3 - (2x - 3)$$

$$\frac{x}{2} + 1 = 3 - 2x + 3$$

$$\frac{5}{2}x = 5$$

$$x = 2$$

Hence, $\left|\frac{x}{2}\right| + 1 \leq 3 - |2x - 3|$ for

$$\frac{2}{3} \leq x \leq 2$$

9 Kia is a solving a logarithmic equation and her solution is shown below.

Line 1: $3\log_3(x) - \log_3\left(\sqrt{x}\right) = 10$

Line 2: $3\log_3\left(\frac{x}{\sqrt{x}}\right) = 10$

Line 3: $3\log_3\left(x^{\frac{1}{2}}\right) = 10$

Line 4: $\frac{3}{2}\log_3(x) = 10$

Line 5: $\log_3(x) = \frac{20}{3}$

Line 6: $x = \left(\frac{20}{3}\right)^3$

Line 7: $x = \frac{8000}{27}$

a) Identify the two mistakes that Kia has made.

[2 marks]

Line 2: She cannot apply the subtraction law as there is a 3 outside of one of the logarithms (multiplying it) but not the over.

Line 6: If $\log_3(x) = a$ then $x = 3^a$ not a^3.

b) Correctly solve this equation.

[3 marks]

$$3\log_3(x) - \log_3\sqrt{x} = 10$$

$$\log_3(x^3) - \log_3(x^{1/2}) = 10$$

$$\log_3\left(\frac{x^3}{x^{1/2}}\right) = 10$$

$$\log_3(x^{5/2}) = 10$$

$$\frac{5}{2}\log_3(x) = 10$$

$$\log_3(x) = \frac{10\times 2}{5}$$

$$\log_3(x) = 4$$

$$x = 3^4$$

$$x = 81$$

10 A sequence of triangular tiles is to be cut from a sheet of ABS plastic.

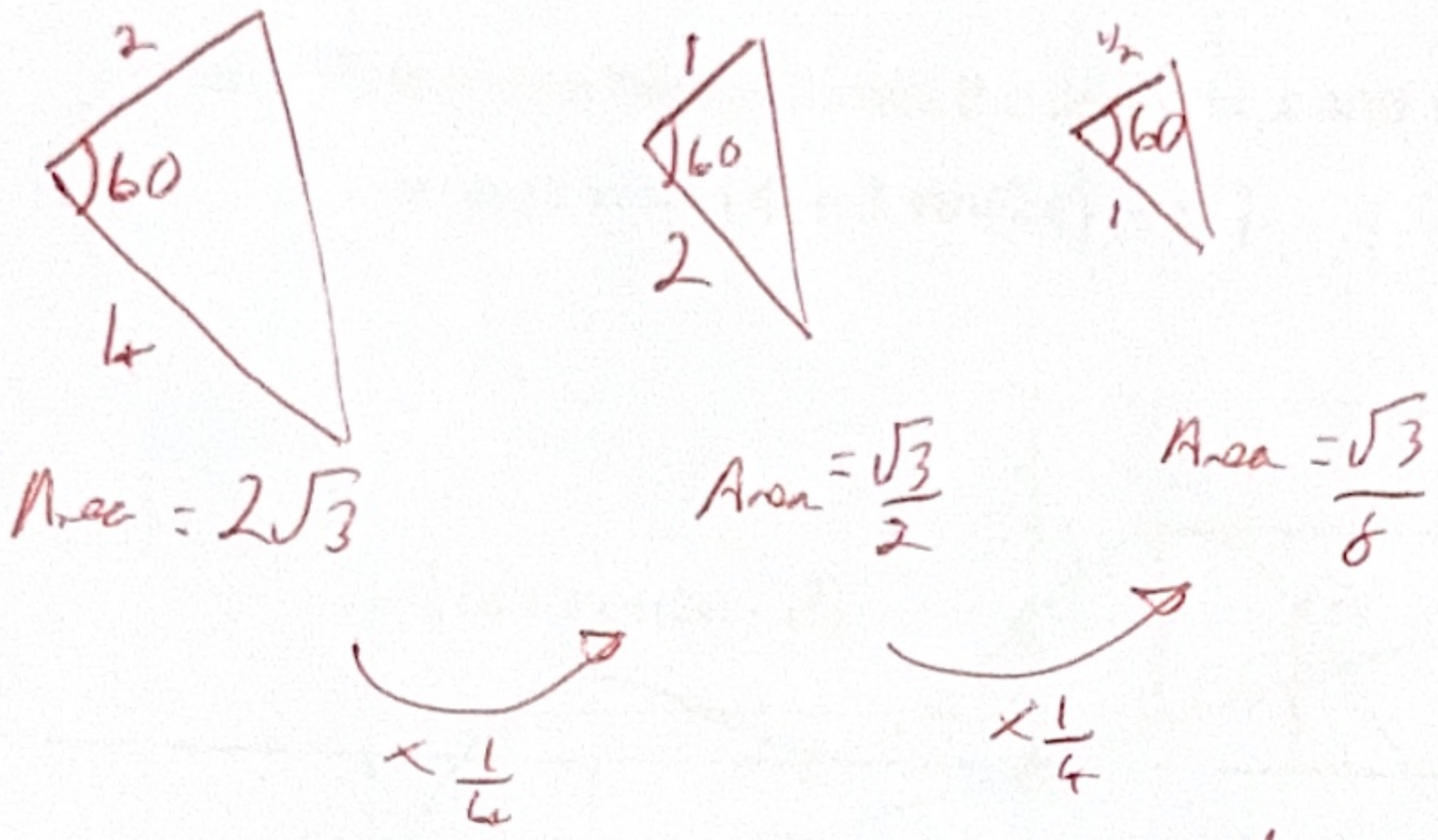
The first tile to be cut has two sides of length 4 cm 2 cm respectively with an angle of $60°$ between them.

Subsequent tiles are made by enlarging the previous tile by a scale factor of a $\frac{1}{2}$.

Show that all tiles to be made from this sequence can be cut from a sheet of ABS of area 5 cm^2.

[5 marks]

Tile 1 Tile 2 Tile 3 ...



So a geometric sequence with first term $a = 2\sqrt{3}$ and common ratio $r = \frac{1}{4}$

Since $|r| = \frac{1}{4} < 1$ the sum to infinity exists and converges

$$S_\infty = \frac{a}{1-r} = \frac{2\sqrt{3}}{1-\frac{1}{4}}$$

$$= \frac{8\sqrt{3}}{3}$$

$$\approx 4.6188$$

Since $4.6188 < 5$ all tiles can be cut from a sheet of area 5cm^2

11 a) Given that $x > 0$, show that the curves of $y = x^2 + 2x - 4$ and $y = 3\sin(2x)$ intersect at a point α such that $1 < \alpha < 1.5$.

Consider $f(x) = x^2 + 2x - 4 - 3\sin(2x)$ [2 marks]

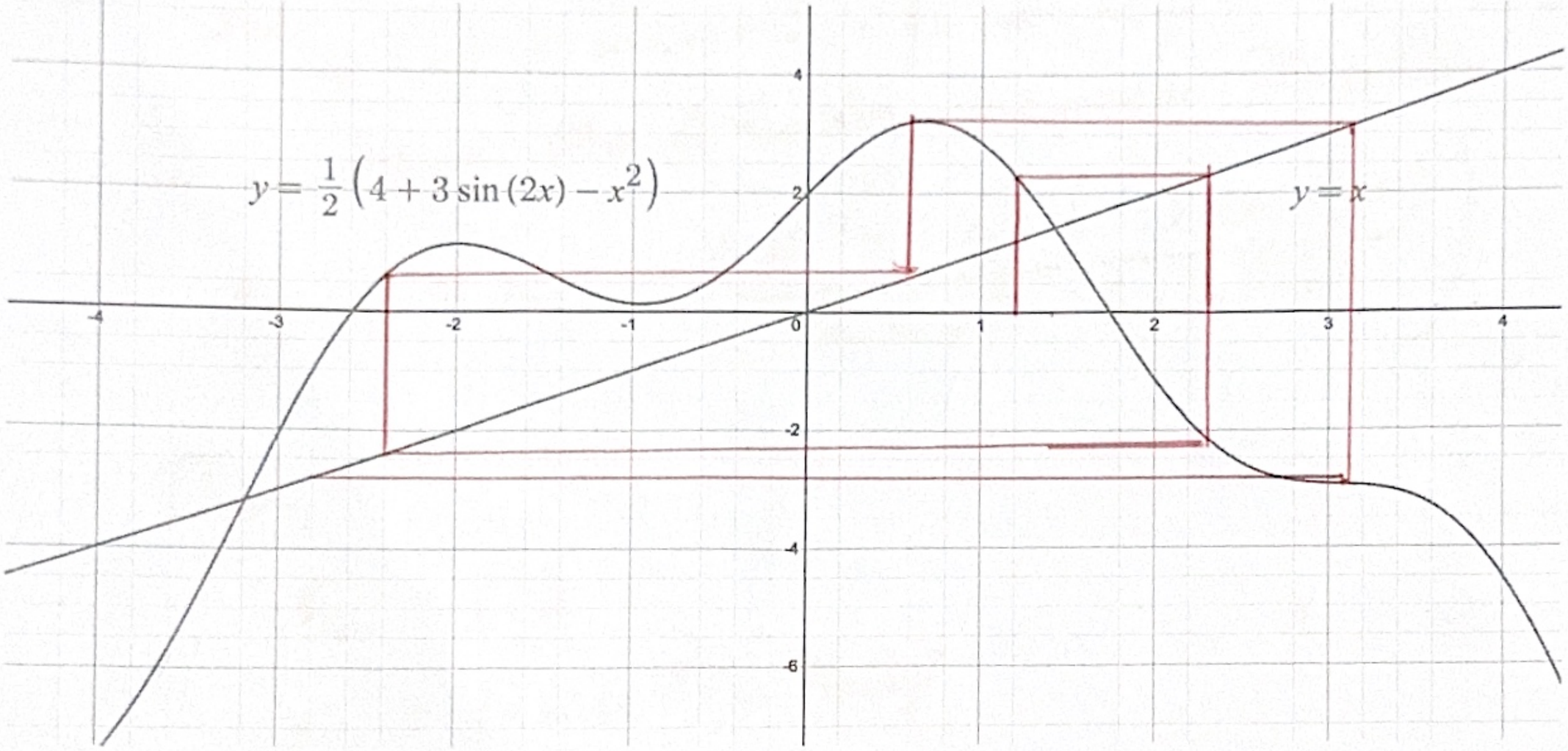
Then

$f(1) = -3$

$f(1.5) =$

Since $f(x)$ is continuous and there is a sign change, the root α lies in the interval $1 < \alpha < 1.5$.

b) The graph below shows the line $y = x$ and a function $y = g(x) = \frac{1}{2}\left(4 + 3\sin(2x) - x^2\right)$.



Using annotations on the graph explain why the iteration defined by

$$x_{n+1} = \frac{1}{2}\left(4 + 3\sin(2x_n) - x_n^2\right)$$

with $x_0 = 1.2$ will not converge to the point $x = \alpha$.

With this iteration and $x_0 = 1.2$ we obtain a diverging cobweb diagram and so we don't converge to α. [2 marks]

c) Gerry uses a Newton-Raphson iteration with a starting value of $x_0 = 1.2$ to find α.

Form the iteration and perform three iterates to find x_3, a better approximation to $x = \alpha$.

[4 mark]

Let $f(x) = x^2 + 2x - 4 - 3\sin(2x)$

then $f'(x) = 2x + 2 - 6\cos(2x)$

So we have the Newton-Raphson iteration

$$x_{n+1} = x_n - \left(\frac{x^2 + 2x - 4 - 3\sin(2x)}{2x + 2 - 6\cos(2x)} \right)$$

Then,

$x_0 = 1.2$

$x_1 = 1.44477676427$

$x_2 = 1.42342560Z$

$x_3 = 1.42328258 6$

So $x_3 = 1.4233$ to 4dp

d) Show that to three decimal places the root is $\alpha = 1.423$

Again, let $f(x) = x^2 + 2x - 4 - 3\sin(2x)$ [2 marks]

Then $f(1.4225) = -0.0082837$

$f(1.4235) = 0.00230202171$

Since $f(x)$ is continuous and $f(1.4225) < 0$ and $f(1.4235) > 0$, to 3dp the root is $\alpha = 1.423$.

e) Katie uses the same Newton-Raphson iteration but with a starting value of $x_0 = 0.52$.

Katie observes that it takes her longer to converge to the root than Tom.

Explain why this may be the case.

[2 marks]

$f'(0.52) = 0.002678456$.

Here, 0.52 is close to a stationary point and so the gradient of the tangent will initially be very shallow, taking you further away from the root before converging

12 The scaled concentration, Z, of a reagent in a chemical equation at time, t hours, can be modelled by the differential equation.

$$\frac{dZ}{dt} = \frac{e^{-2Z}(4t + 11)}{t^2 + 5t + 6}$$

a) Given that at time $t = 0$ the scaled concentration is $Z = 2$ grams per litre, find a solution of the form

$$Z = \frac{1}{a} \ln\left(2 \ln\left(\frac{f(t)}{b}\right) + e^d\right)$$

where $f(t)$ is some function of t to be found and a, b and d are i integer constants.

[8 marks]

$$\frac{dZ}{dt} = \frac{e^{-2Z}(4t+11)}{t^2+5t+6}$$

$$\Rightarrow \int \frac{1}{e^{-2Z}}\, dZ = \int \frac{4t+11}{t^2+5t+6}$$

$$\int e^{2Z}\, dZ = \int \frac{3}{t+2} + \frac{1}{t+3}\, dt$$

So

$$e^{2Z} = 3\ln|t+2| + \ln|t+3| + k$$

$$\frac{e^4}{2} = 3\ln|2| + \ln|3| + k$$

$$\Rightarrow k = \frac{e^4}{2} - (\ln|2^3| + \ln|3|) = \frac{e^4}{2} - \ln|24|$$

when $t=0$, $Z=2$, so

Hence,

$$\frac{e^2}{2} = 3\ln|t+2| + \ln|t+3| + \frac{e^4}{2} - \ln|24|$$

$$= \ln\left|\frac{(t+2)^3(t+3)}{24}\right| + \frac{e^4}{2}$$

So,

$$e^{2z} = 2\ln\left|\frac{(t+2)^3(t+3)}{24}\right| + e^4$$

$$2z = \ln\left(2\ln\left|\frac{(t+2)^3(t+3)}{24}\right| + e^4\right)$$

Hence,

$$z = \frac{1}{2}\ln\left(2\ln\left|\frac{(t+2)^3(t+3)}{24}\right| + e^4\right)$$

b) What is the scaled concentration after 2 hours.

When $t=2$,

[2 marks]

$$z = \frac{1}{2}\ln\left(2\ln\left|\frac{4^3\times 5}{24}\right| + e^4\right)$$

$= 2.045$ grams per litre to 3dp

13 Let $y = f(x) = e^{-2x^2+12x}$ be a curve in Cartesian space.

a) Find the x—coordinates of the points of inflection of the curve

[5 marks]

Let $y = e^{-2x^2+12x}$

Then,

$$\frac{dy}{dx} = e^{-2x^2+12x}(-4x+12)$$

Let $u = e^{-2x^2+12x}$

$$\frac{du}{dx} = (-4x+12)e^{-2x^2+12x}$$

and $v = -4x+12$

$$\frac{dv}{dx} = -4$$

So, by the product rule,

$$\frac{d^2y}{dx^2} = -4e^{-2x^2+12x} + (-4x+12)(-4x+12)e^{-2x^2+12x}$$

$$= e^{-2x^2+12x}(-4 + 16x^2 - 96x + 144)$$

$$= e^{-2x^2+12x}(16x^2 - 96x + 140)$$

$$= 4e^{-2x^2+12x}(4x^2 - 24x + 35)$$

$$= 4e^{-2x^2+12x}(2x-7)(2x-5)$$

At an inflection point $\frac{d^2y}{dx^2} = 0$. so inflection points at $x = \frac{7}{2}$ and $x = \frac{5}{2}$

b) Hence, identify the regions where $f(x)$ is concave and convex.

[2 marks]

Concave if $\frac{d^2y}{dx^2} < 0$, so $f(x)$ is concave

for $\frac{5}{2} < x < \frac{7}{2}$

Convex if $\frac{d^2y}{dx^2} > 0$, so $f(x)$ is convex

when $x < \frac{5}{2}$ or $x > \frac{7}{2}$

14 a) Prove that $\sin(3\theta) = 3\sin(\theta) - 4\sin^3(\theta)$

[4 marks]

Using the compound angle identities,

$\sin(3\theta) = \sin(2\theta + \theta)$

$= \sin(2\theta)\cos(\theta) + \cos(2\theta)\sin(\theta)$

$= 2\sin(\theta)\cos(\theta)\cos(\theta) + (1 - 2\sin^2(\theta))\sin(\theta)$

$= 2\sin(\theta)\cos^2(\theta) + \sin(\theta) - 2\sin^3(\theta)$

$= 2\sin(\theta)(1 - \sin^2(\theta)) + \sin(\theta) - 2\sin^3(\theta)$

$= 2\sin(\theta) - 2\sin^3(\theta) + \sin(\theta) - 2\sin^3(\theta)$

$= 3\sin(\theta) - 4\sin^3(\theta)$

b) Using part (a) and the given identity $\cos(3\theta) = 4\cos^3(\theta) - 3\cos(\theta)$ show that

$$\tan(3\theta) = \frac{a\tan(\theta) - b\tan^3(\theta)}{b - a\tan^2(\theta)}$$

where $a, b \in \mathbb{Z}$ are to be found.

[5 marks]

$$\tan(3\theta) = \frac{\sin(3\theta)}{\cos(3\theta)}$$

$$= \frac{3\sin(\theta) - 4\sin^3(\theta)}{4\cos^3(\theta) - 3\cos(\theta)}$$

$$= \frac{\frac{3\sin(\theta)}{4\cos^3(\theta)} - \frac{4\sin^3(\theta)}{4\cos^3(\theta)}}{\frac{4\cos^3(\theta)}{4\cos^3(\theta)} - \frac{3\cos(\theta)}{4\cos^3(\theta)}}$$

$$= \frac{\frac{3\sin(\theta)}{4\cos^3(\theta)} - \tan^3(\theta)}{1 - \frac{3}{4}\frac{\cos\theta}{\cos^3\theta}}$$

$$= \frac{\frac{3}{4}\left(\frac{\sin(\theta)}{\cos^3(\theta)}\right) - \tan^3(\theta)}{1 - \frac{3}{4\cos^2\theta}}$$

$$= \frac{\frac{3}{4}\left(\tan(\theta)\sec^2(\theta)\right) - \tan^3(\theta)}{1 - \frac{3}{4}\sec^2(\theta)}$$

$$= \frac{\frac{3}{4}\tan\theta(1+\tan^2\theta) - \tan^3\theta}{1 - \frac{3}{4}(1+\tan^2\theta)}$$

$$= \frac{\frac{3}{4}\tan(\theta) - \frac{1}{4}\tan^3(\theta)}{\frac{1}{4} - \frac{3}{4}\tan^2(\theta)}$$

$$= \frac{3\tan(\theta) - \tan^3(\theta)}{1 - 3\tan^2(\theta)}$$

In required form with $a = 3$, $b = 1$

c) Hence, for, $0 \leq x \leq 2\pi$, solve

$0 \leq x \leq 2\pi$
$0 \leq 3x \leq 6\pi$

$$\frac{9\tan(x) - 3\tan^3(x)}{1 - 3\tan^2(x)} = \sqrt{3}$$

[3 marks]

$$\frac{9\tan(x) - 3\tan^3(x)}{1 - 3\tan^2(x)} = \sqrt{3}$$

$$\Rightarrow \quad 3\left(\frac{3\tan(x) - \tan^3(x)}{1 - 3\tan^2(x)}\right) = \sqrt{3}$$

$$\Rightarrow \quad 3\tan(3x) = \sqrt{3}$$

$$\tan(3x) = \frac{\sqrt{3}}{3}$$

Hence,

$$3x = \frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}, \frac{19\pi}{6}, \frac{25\pi}{6} \text{ and } \frac{31\pi}{6}$$

So,

$$x = \frac{\pi}{18}, \frac{7\pi}{18}, \frac{13\pi}{18}, \frac{19\pi}{18}, \frac{25\pi}{18} \text{ and } \frac{31\pi}{18}$$

15 The centre of a circle, C, lies on the line $y = \frac{1}{3}x + \frac{11}{3}$.

Given that $A(6,9)$ and $B(-8, -5)$ are two points on the circumference of C, determine the values of k for which the line $4x - 3y = k$ is a tangent to the circle C.

[11 marks]

Midpoint of $AB = \left(\frac{6+-8}{2}, \frac{9+-5}{2}\right)$

$= (-1, 2)$

Gradient of $AB = \frac{9--5}{6--8} = \frac{14}{14} = 1$

So gradient of the perpendicular bisector of AB has gradient -1, and passes through $(-1, 2)$.

So,

$y - 2 = -1(x - -1)$

$y - 2 = -x - 1$

$y = -x + 1$

Centre of the circle is at the intersection of $y = -x + 1$ and $y = \frac{1}{3}x + \frac{11}{3}$. Hence,

$-x + 1 = \frac{1}{3}x + \frac{11}{3}$

$\Rightarrow \quad \frac{4x}{3} = \frac{-8}{3}$

$\Rightarrow \quad x = -2$

When $x = -2$, $y = -2 + 1 = 3$

So the centre of the circle is $(-2, 3)$

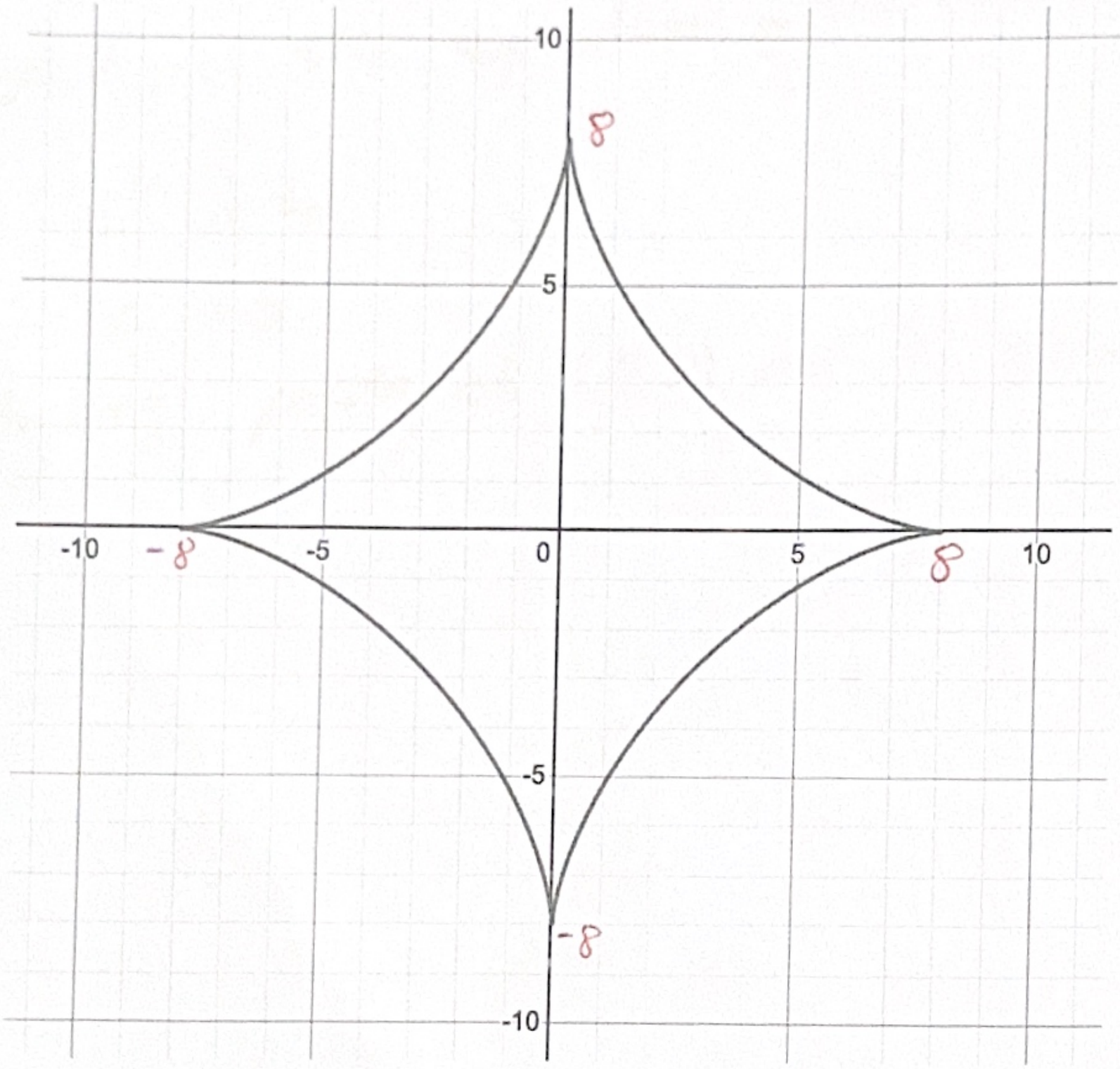
Using the point A and the centre we can find the radius:

$\Rightarrow -36(k+67)(k-33) = 0$

So $4x-3y = -67$ and $4x-3y = 33$ are both tangents to the circle.

17 The curve shown in the picture below is known as an Astroid and has Cartesian equation

$$x^{\frac{2}{3}} + y^{\frac{2}{3}} = 4$$



a) Find thee stationary points of the curve.

[5 marks]

Differentiating implicitly with respect to x,

$$\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{\frac{2}{3}x^{-1/3}}{\frac{2}{3}y^{-1/3}}$$

$$= -\frac{y^{1/3}}{x^{1/3}}$$

At a stationary point $\frac{dy}{dx} = 0$, so

$$-y^{1/3} = 0 \quad \Rightarrow y = 0$$

when $y = 0$, in the equation of the ostroid we have

$$x^{2/3} = 4$$

$$x = 8$$

So stationary points at $(8, 0)$ and by symmetry $(-8, 0)$

b) Given that the parametric form of the astroid shown is $x = 8\cos^3(t), y = 8\sin^3(t)$ for $0 \leq t \leq 2\pi$ find the area enclosed by the curve.

You may use, without proof, that $\int_0^{\frac{\pi}{2}} \sin^6(x)\ dx = \frac{5\pi}{32}$.

Fully justify your answer.

[9 marks]

When $t=0$, $x = 8\cos^3(0) = 8$ and $y = 8\sin^3(0) = 0$

when $t=\frac{\pi}{2}$, $x = 8\cos^3(\frac{\pi}{2}) = 0$ and $y = 8\sin^3(\frac{\pi}{2}) = 8$.

Hence, if we find the area between $t=0$ and $t=\frac{\pi}{2}$ and then multiply by 4 we have the area enclosed by the curve.

Consider $\int_0^{\pi/2} y\frac{dx}{dt}\,dt$.

with $x = 8\cos^3(t)$, $\frac{dx}{dt} = 24\cos^2(t) \times -\sin(t)$

$= -24\cos^2(t)\sin(t)$

So, Area of $\frac{1}{4}$ curve is

$$A = \int_0^{\pi/2} 8\sin^3(t) \times -24\cos^2(t)\sin(t)$$

$$= -192\int_0^{\pi/2} \sin^3(t)\cos^2(t)\sin(t)\,dt$$

$$= -192\int_0^{\pi/2} \sin^4(t)\cos^2(t)\,dt$$

$$= -192 \int_0^{\pi/2} \sin^4(t)\left(1 - \sin^2(t)\right) dt$$

$$= -192 \int_0^{\pi/2} \sin^4(t) - \sin^6(t)\, dt$$

Since $\int_0^{\pi/2} \sin^6(t)\, dt = \frac{5\pi}{32}$, lets calculate

$$\int_0^{\pi/2} \sin^4(t)\, dt = \int_0^{\pi/2} \left(\sin^2(t)\right)^2 dt$$

$$= \int_0^{\pi/2} \left(\frac{1 - \cos(2t)}{2}\right)^2 dt$$

$$= \int_0^{\pi/2} \frac{1 - 2\cos(2t) + \cos^2(t)}{4}\, dt$$

$$= \int_0^{\pi/2} \frac{1}{4} - \frac{1}{2}\cos(2t) + \frac{1}{4}\left(\frac{\cos(4t) + 1}{2}\right) dt$$

$$= \int_0^{\pi/2} \frac{1}{4} - \frac{1}{2}\cos(2t) + \frac{1}{8} + \frac{1}{8}\cos(4t)\, dt$$

$$= \int_0^{\pi/2} \frac{3}{8} - \frac{1}{2}\cos(2t) + \frac{1}{8}\cos(4t)\, dt$$

$$= \left[\frac{3}{8}t - \frac{1}{4}\sin(2t) + \frac{1}{32}\sin(4t)\right]_0^{\pi/2}$$

$= \left(\frac{3\pi}{16} - 0 - 0\right) - \left(0 + 0 + 0\right)$

$= \frac{3\pi}{16}$

5. $A = -192\left[\frac{3\pi}{16} - \frac{5\pi}{32}\right]$

$= -192 \times \frac{1}{32}\pi$

$= -6\pi$.

So area enclosed by the curve is $4 \times 6\pi = 24\pi$

$$r = \sqrt{(6--2)^2 + (9-5)^2}$$
$$= \sqrt{8^2 + 6^2}$$
$$= \sqrt{100}$$
$$= 10$$

So, equation of circle is,

$$(x+2)^2 + (y-3)^2 = 100 \quad (1)$$

Consider $4x - 3y = k \Rightarrow y = \frac{4x-k}{3}$ (2)

Substitute (2) into (1)

$$(x+2)^2 + \left(\frac{4x-k}{3} - 3\right)^2 = 100$$

$$\Rightarrow x^2 + 4x + 4 + \frac{1}{9}(4x-k-9)^2 = 100$$

$$\Rightarrow 9x^2 + 36x + 36 + (16x^2 - 8kx - 72x + k^2 + 18k + 81) = 900$$

$$\Rightarrow 25x^2 + (-36 - 8k)x + (k^2 + 18k - 783) = 0 \quad (\dagger)$$

For $4x - 3y = k$ to be a tangent to the circle, the discriminant of $(\dagger)$ is 0.

Hence,

$$(-36-8k)^2 - 4 \times 25 \times (k^2 + 8k - 783) = 0$$

$$\Rightarrow 1296 + 576k + 64k^2 - 100k^2 - 1800k + 78300 = 0$$

$$\Rightarrow -36k^2 - 1224k + 79596 = 0$$

$$-36(k^2 - 34k + 2211) = 0$$