

## AQA A-Level Maths 2026 Paper 2

Do not turn over the page until instructed to do so.

This assessment is out of 100 marks and you will be given 120 minutes.

When you are asked to by your teacher write your **full name** below

**Name:**

**Total Marks:**                      / 100

## Section A

- 1 The equation  $\sin(2x) = \frac{1}{5}$  has two solutions in the range  $0 \leq x \leq 2\pi$ .

How many solutions has the equation

$$\sin(6x) = \frac{1}{5}, \quad 0 \leq x \leq 6\pi?$$

- 12                                  6                                  72                                  36

$6 \times 2 = 12$  solutions between 0 and  $2\pi$ . [1 mark]  
 So  $12 \times 3 = 36$  solutions in the interval.

- 2 Given that  $\int_0^2 f(x) dx = 6$  what is the value of  $\int_2^8 f\left(\frac{x-2}{3}\right) dx$ ?

- 6                                  2                                  18                                  8

[1 mark]

Let  $u = \frac{x-2}{3}$

$$\frac{du}{dx} = \frac{1}{3} \Rightarrow dx = 3du$$

when  $x = 8$ ,  $u = 2$

$x = 2$ ,  $u = 0$

$$\begin{aligned} \text{So } \int_2^8 f\left(\frac{x-2}{3}\right) dx &= \int_0^2 f(u) 3du = 3 \int_0^2 f(u) du \\ & \underbrace{\hspace{10em}}_{\text{same as original}} \\ &= 3 \times 6 = 18 \end{aligned}$$

- 3 What is the Cartesian equation for the curve with parametric equations:  $x = 3 + 2 \cos(\theta)$ ,  $y = 4 + 3 \sin(\theta)$  ?

$$9(x - 3)^2 + 4(y - 4)^2 = 36$$

$$9(x - 3)^2 - 4(y - 4)^2 = 1$$

$$4(x - 3)^2 + 9(y - 4)^2 = 36$$

$$4(x - 3)^2 + 9(y - 4)^2 = 36$$

$$x = 3 + 2 \cos(\theta)$$

$$y = 4 + 3 \sin(\theta)$$

$$\Rightarrow \frac{x-3}{2} = \cos \theta$$

$$\Rightarrow \frac{y-4}{3} = \sin \theta$$

$$\sin^2 \theta + \cos^2 \theta = 1 \Rightarrow \left(\frac{y-4}{3}\right)^2 + \left(\frac{x-3}{2}\right)^2 = 1$$

$$\Rightarrow 4(y-4)^2 + 9(x-3)^2 = 36$$

[1 mark]

- 4 Let  $u_r = 3 + 2r$  be the  $r$ th term of a sequence.

Find  $\sum_{r=5}^{20} u_r$

480

448

435

416

[1 mark]

$$\sum_{r=5}^{20} u_r = \sum_{r=1}^{20} u_r - \sum_{r=1}^4 u_r$$

- 5 Prove that the sum of the squares of two consecutive positive even integers is a multiple of 4.

for  $k \in \mathbb{N}$

Let  $2k$  and  $2k+2$  be the two consecutive even numbers. Then [3 marks]

$$\begin{aligned} (2k)^2 + (2k+2)^2 &= 4k^2 + 4k^2 + 8k + 4 \\ &= 8k^2 + 8k + 4 \\ &= 4(2k^2 + 2k + 1) \end{aligned}$$

which is a multiple of 4 since  $2k^2 + 2k + 1$  is an integer

6 In the expansion of  $(a + bx)^5$  the following holds

- the coefficient of the  $x^4$  term is 810,
- the coefficient of the  $x^2$  term is three times that of the coefficient of the  $x$  term.

Find the values of the constants  $a$  and  $b$ .

[5 marks]

Coefficient of  $x^4 = 810$

$$\Rightarrow \binom{5}{4} a (b)^4 = 810$$

$$5ab^4 = 810$$

$$ab^4 = 162 \quad (1)$$

Coefficient of  $x^2$  is three times that of the  $x$  term

$$\binom{5}{2} a^3 b^2 = 3 \times \binom{5}{1} a^4 b$$

$$10a^3 b^2 = 15a^4 b$$

$$\Rightarrow 10b = 15a \quad (2)$$

$$(2) \Rightarrow \frac{2}{3} b = a$$

$$\text{Sub into (1)} \quad \frac{2}{3} b \times b^4 = 162 \Rightarrow b^5 = \frac{3 \times 162}{2}$$

$$\Rightarrow b^5 = 243$$

$$\Rightarrow b = 3$$

Now, using (2),  $a = 2$

Well done :)

7 The circle  $C$  has equation  $x^2 - 4x + y^2 - 6y - 12 = 0$ .

$P$  and  $Q$  are two points on the circumference of this circle, such that the midpoint of  $PQ$  has coordinates  $M(1,5)$ .

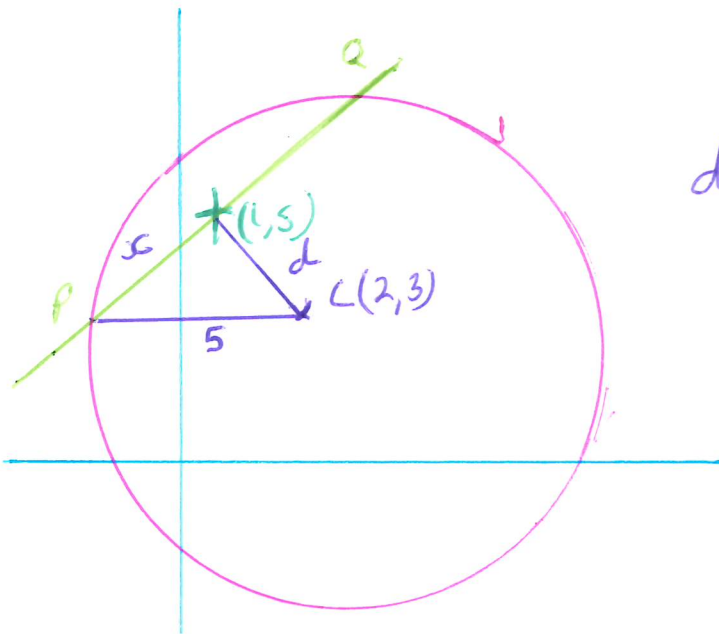
Find, in exact form, the length of the chord  $PQ$ .

[6 marks]

$$x^2 - 4x + y^2 - 6y - 12 = 0$$

$$\Rightarrow (x-2)^2 + (y-3)^2 = 25$$

Centre of  $C = (2,3)$   
radius of  $C = 5$



$$d = \sqrt{(1-2)^2 + (5-3)^2}$$

$$= \sqrt{5}$$

Hence,

$$x = \sqrt{5^2 - (\sqrt{5})^2}$$

$$= \sqrt{25 - 5}$$

$$= \sqrt{20}$$

$$= 2\sqrt{5}$$

So, in exact form length  $PQ = 2 \times 2\sqrt{5} = 4\sqrt{5}$  as  $M$  is the midpoint of  $PQ$ .



8 Show that

$$\int_2^7 \frac{3x+14}{x+4} dx = a + \ln\left(\frac{b^2}{c^2}\right)$$

where  $a$ ,  $b$  and  $c$  are integers to be found and stated.

[5 marks]

$$\frac{3x+14}{x+4} = 3 \frac{(x+4)+2}{x+4} = 3 + \frac{2}{x+4}$$

[for polynomial division]

$$\text{Hence, } \int_2^7 \frac{3x+14}{x+4} dx = \int_2^7 3 + \frac{2}{x+4} dx$$

$$= \left[ 3x + 2 \ln|x+4| \right]_2^7$$

$$= \left[ 21 + 2 \ln|11| \right] - \left( 6 + 2 \ln|6| \right)$$

$$= 15 + \ln|121| - \ln|36|$$

$$= 15 + \ln \left| \frac{121}{36} \right|$$

$$= 15 + \ln \left| \frac{11^2}{2^2 \cdot 3^2} \right|$$

so,  $a = 15$ ,  $b = 11$ ,  $c = 6$ .

- 9 The function  $f$  is defined by

$$f(x) = \frac{3x + 4}{x + 2}$$

- a) State the maximum possible domain of  $f(x)$

[1 mark]

$$x \in \mathbb{R}, x \neq -2$$

or

$$x \in \mathbb{R} \setminus \{-2\}$$

- b) (i) Without explicitly finding  $f^{-1}(x)$  and fully justifying your answer find the value of  $a$  such that  $f^{-1}(a) = 7$

[2 marks]

$$\begin{aligned}
 f^{-1}(a) &= 7 \\
 \Rightarrow f(f^{-1}(a)) &= f(7) \\
 a &= f(7) \\
 &= \frac{3 \times 7 + 4}{7 + 2} \\
 &= \frac{25}{9}
 \end{aligned}$$

(ii) Find  $f^{-1}(x)$ 

[3 marks]

$$\text{Let } y = \frac{3x+4}{x+2}$$

$$\Rightarrow yx + 2y = 3x + 4$$

$$yx - 3x = 4 - 2y$$

$$x(y-3) = 4-2y$$

$$x = \frac{4-2y}{y-3}$$

Then,

$$f^{-1}(x) = \frac{4-2x}{x-3}$$

(iii) Explain how the domain of  $f^{-1}(x)$  tells you that  $y = 3$  is an asymptote of  $f(x)$ .

[1 mark]

The domain of  $f^{-1}(x)$  is  $x \in \mathbb{R}, x \neq 3$ .

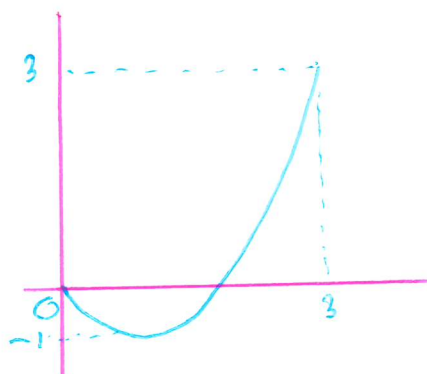
This is the range of  $f(x)$ , so

~~$y \neq 3$  is the range~~

$y = 3$  is an asymptote

Let  $g(x) = x^2 - 2x$ ,  $x \in \mathbb{R}$ ,  $0 \leq x \leq 3$

- c) (i) Find the range of  $g(x)$  giving your answer in set notation.



[2 marks]

So  
 $-1 \leq g(x) \leq 3$

So the range in set notation is

$$\{g(x) : -1 \leq g(x) \leq 3, g(x) \in \mathbb{R}\}$$

- (ii) Explain why  $g(x)$  cannot have an inverse.

[1 mark]

$g(x)$  does not pass the horizontal line test and so is many-to-one.  
 Hence,  $g(x)$  cannot have an inverse.

- d) Given that  $fg(x) = \frac{3x^2 - 6x + 4}{x^2 - 2x + 2}$ , show that the solutions of  $fg(x) = gf(x)$  satisfy,

$$8x^3 - 6x^2 - 16x + 16 = 0$$

[5 marks]

$$fg(x) = f(x^2 - 2x) = \frac{3x^2 - 6x + 4}{x^2 - 2x + 2} \text{ in question}$$

$$\begin{aligned} gf(x) &= g\left(\frac{3x+4}{x+2}\right) \\ &= \left(\frac{3x+4}{x+2}\right)^2 - 2\left(\frac{3x+4}{x+2}\right) \\ &= \frac{(3x+4)^2 - 2(3x+4)(x+2)}{(x+2)^2} \\ &= \frac{9x^2 + 24x + 16 - 6x^2 - 20x + 4}{(x+2)^2} \\ &= \frac{3x^2 + 4x}{(x+2)^2} \end{aligned}$$

Now

$fg(x) = gf(x)$  is the same as

$$\frac{3x^2 - 6x + 4}{x^2 - 2x + 2} = \frac{3x^2 + 4x}{(x+2)^2}$$

$$\Rightarrow (3x^2 - 6x + 4)(x+2)^2 = (3x^2 + 4x)(x^2 - 2x + 2)$$

$$\Rightarrow (3x^2 - 6x + 4)(x+2)^2 = (3x^2 + 4x)(x^2 - 2x + 2)$$

$$\begin{aligned} \Rightarrow 3x^4 + 12x^3 + 12x^2 - 6x^3 - 24x^2 - 24x + 4x^3 + 16x + 16 \\ = 3x^4 - 2x^3 - 2x^2 + 8x \end{aligned}$$

$$\Rightarrow 3x^4 + 6x^3 - 12x^2 - 8x + 16 = 3x^4 - 2x^3 - 2x^2 + 8x$$

$$\Rightarrow 8x^3 - 6x^2 - 16x + 16 = 0 \text{ as required}$$

- 10 a) Find  $\frac{dy}{dx}$  for the curve with equation

$$2x^2 + 4y^3 - 8xy = 0$$

Differentiating w.r.t  $x$

[3 marks]

$$4x + 12y^2 \frac{dy}{dx} - 8 \left[ x \frac{dy}{dx} + y \right] = 0$$

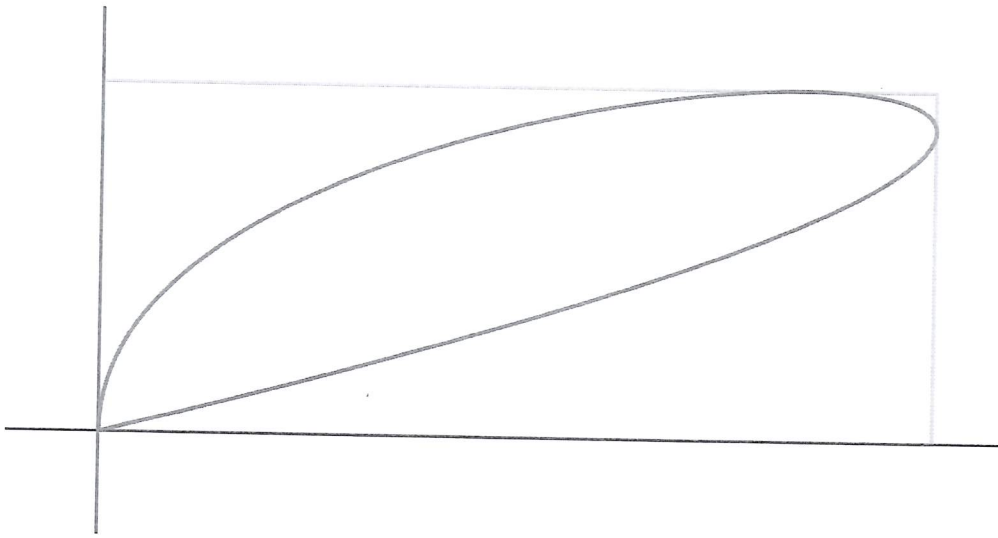
$$4x - 8y = 8x \frac{dy}{dx} - 12y^2 \frac{dy}{dx}$$

$$\Rightarrow 4x - 8y = \frac{dy}{dx} (8x - 12y^2)$$

$$\begin{aligned} \Rightarrow \frac{dy}{dx} &= \frac{4x - 8y}{8x - 12y^2} \\ &= \frac{x - 2y}{2x - 3y^2} \end{aligned}$$

- b) The image below shows a sketch of the curve with equation

$$2x^2 + 4y^3 - 8xy = 0, \text{ such that } x \geq 0, y \geq 0.$$



Around the curve is placed a bounding box. This bounding box is the smallest rectangle of the shown orientation that totally encloses the curve in the positive quadrant.

Show that the area of the box is  $\frac{256}{27}$  square units.

Maximum when  $\frac{dy}{dx} = 0$

$$\Rightarrow x - 2y = 0$$

$$\Rightarrow x = 2y$$

but this also satisfies the original equation, so

$$2(2y)^2 + 4y^3 - 8(2y)(y) = 0$$

$$\Rightarrow 8y^2 + 4y^3 - 16y^2 = 0$$

$$4y^3 - 8y^2 = 0$$

$$4y^2(y - 2) = 0$$

$$\text{So } y = 0, \text{ or } y = 2$$

Maximum  $x$  coordinate when  $\frac{dx}{dy} = 0$ ,

i.e.,  $2x - 3y^2 = 0$

$$\Rightarrow y^2 = \frac{2x}{3}$$

So, into original equation

$$2x^2 + 4\left(\frac{2x}{3}\right)^{3/2} - 8x\left(\frac{2x}{3}\right)^{1/2} = 0$$

$$\Rightarrow 2x^2 + \frac{8x}{3}\left(\frac{2x}{3}\right)^{1/2} - 8x\left(\frac{2x}{3}\right)^{1/2} = 0$$

$$\Rightarrow 2x\left(x - \frac{8}{3}\left(\frac{2x}{3}\right)^{1/2}\right) = 0$$

So  $x = 0$  or  $x - \frac{8}{3}\left(\frac{2x}{3}\right)^{1/2} = 0$

$$x = \frac{8}{3}\left(\frac{2x}{3}\right)^{1/2}$$

$$x^{1/2} = \frac{8}{3}\left(\frac{2}{3}\right)^{1/2}$$

$$x = \frac{128}{27}$$

Hence, area of bounding box is

$$2 \times \frac{128}{27}$$

$$= \frac{256}{27}$$



## Section B

- 11 A particle is moving with constant velocity under the action of three forces  $\mathbf{F}_1$ ,  $\mathbf{F}_2$  and  $\mathbf{F}_3$ . Given that  $\mathbf{F}_1 = 2\mathbf{i} + 5\mathbf{j}$  N and  $\mathbf{F}_2 = -4\mathbf{i} + 3\mathbf{j}$ . What is  $\mathbf{F}_3$ ?

$2\mathbf{i} + 8\mathbf{j}$

$2\mathbf{i} - 8\mathbf{j}$

$-6\mathbf{i} + 2\mathbf{j}$

$-2\mathbf{i} + 8\mathbf{j}$

Constant velocity  $\Rightarrow a = 0$

[1 mark]

So ~~in~~ ~~equilibrium~~ resultant force = 0

$$\begin{pmatrix} 2 \\ 5 \end{pmatrix} + \begin{pmatrix} -4 \\ 3 \end{pmatrix} + \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ -8 \end{pmatrix}$$

- 12 In this question use  $g = 10 \text{ ms}^{-2}$

A box of mass 3.2 kg is pulled across a rough horizontal surface by a force of magnitude 40 N. The force acts parallel to the surface.

Given that the box is moving with an acceleration of  $6 \text{ ms}^{-2}$ , what is the coefficient of friction,  $\mu$ ?

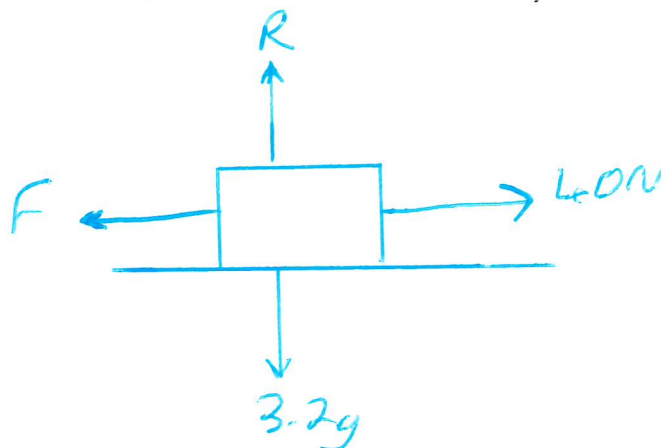
$\mu = 0.65$

$\mu = 1.53$

$\mu = 0.66$

$\mu = 1.25$

[1 mark]



$F = \mu R$

$$R \uparrow \quad R - 3.2g = 0 \Rightarrow R = 3.2g$$

NB:  $4g = 9.8$  then  
 $\mu = 0.66$ .

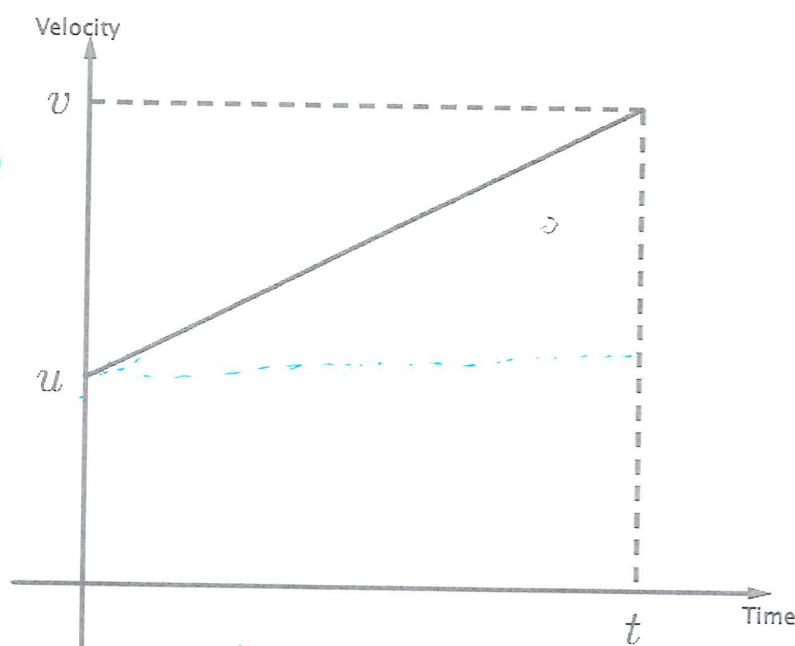
Apply  $F = ma \rightarrow$

$$40 - F = 3.2a$$

$$40 - \mu R = 3.2 \times 6$$

$$20.8 = \mu \times 3.2g \Rightarrow \mu = 0.65$$

- 13 Using the velocity time graph shown below derive the SUVAT equations  $s = ut + \frac{1}{2}at^2$  and  $v^2 = u^2 + 2as$ .



Displacement is area under the graph  
acceleration is the gradient

Area:

$$s = \left(\frac{u+v}{2}\right)t \quad (1)$$

Gradient  $a = \frac{v-u}{t}$

$$\Rightarrow t = \frac{v-u}{a} \quad (2)$$

So

$$s = \left(\frac{u+v}{2}\right)\left(\frac{v-u}{a}\right)$$

$$2as = uv + v^2 - uv - u^2$$

$$\Rightarrow v^2 = u^2 + 2as$$

area of rectangle + triangle

[4 marks]

and

$$s = ut + \frac{1}{2}t(v-u)$$

but from (2),  $v-u = at$

So

$$s = ut + \frac{1}{2}t \cdot at$$

$$= ut + \frac{1}{2}at^2$$

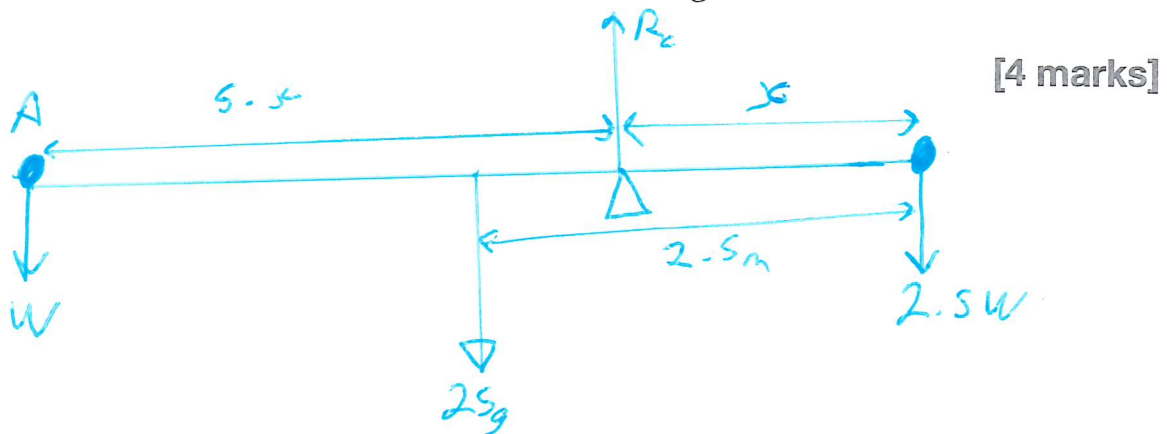
- 14 A straight uniform beam,  $AB$ , of length 5 m has mass 25 kg.

A particle of weight  $W$  newtons is attached at  $A$  and a particle of weight  $\frac{5}{2}W$  is attached at  $B$ .

This system is then placed on a pivot  $C$  at a distance  $x$  metres from  $B$  and rests in equilibrium with  $AB$  horizontal.

Show that

$$x = \frac{5W + 62.5g}{3.5W + 25g}$$



$$\begin{aligned} \text{A)} \quad W(5-x) + 25g(2.5-x) - 2.5Wx &= 0 \quad \text{as in equilibrium} \\ 5W - xW + 62.5g - 25gx - 2.5Wx &= 0 \end{aligned}$$

$$\Rightarrow \quad 5W + 62.5g = 2.5Wx + 25gx + Wx$$

$$5W + 62.5g = x(3.5W + 25g)$$

$$x = \frac{5W + 62.5g}{3.5W + 25g}$$



- 15 A ball of mass 600g is moving along a flat, smooth, horizontal surface with a velocity that depends on time given by

$$v(t) = -0.2t^2 + 5t + 3$$

- a) Find the time at which it's velocity is maximum and the magnitude of the force acting on the ball at this time.

[3 marks]

$$\frac{dv}{dt} = 0 \text{ at the maximum velocity}$$

$$\frac{dv}{dt} = -0.4t + 5$$

Hence, maximum when

$$-0.4t + 5 = 0$$

$$5 = 0.4t$$

$$t = \frac{5}{0.4}$$

$$= 12.5 \text{ s}$$

At maximum velocity  $\frac{dv}{dt} = a = 0$ , so

using  $F = ma$ ,

$$|F| = 0.$$

- b) Find the displacement of the ball, relative to where it started at time  $t = 20$  seconds.

[3 marks]

$$v(t) = -0.2t^2 + 5t + 3$$

$$\text{so } s(t) = \int_0^t v(t) dt$$

$$= -\frac{1}{15}t^3 + \frac{5}{2}t^2 + 3t + C$$

At  $t=0$ ,  $s=0$ , so  $C=0$

$$\text{Hence, } s(t) = -\frac{1}{15}t^3 + \frac{5}{2}t^2 + 3t$$

and

$$s(20) = -\frac{1}{15} \times 20^3 + \frac{5}{2} \times 20^2 + 3 \times 20$$

$$= \frac{1580}{3} \text{ m}$$

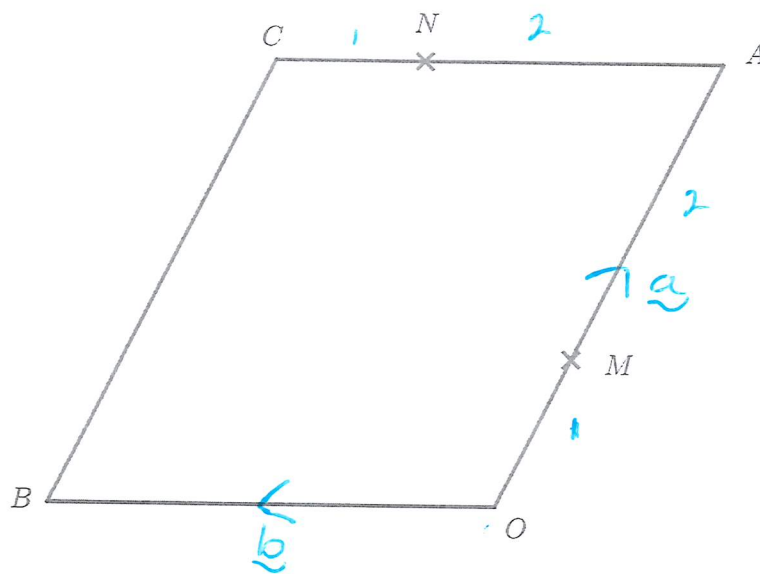
$$= 527 \text{ m to 3 sig fig}$$

- 16 The picture below represents a network of paths in a wildflower meadow.

$OACB$  is a parallelogram and  $\overrightarrow{OA} = \mathbf{a}$  and  $\overrightarrow{OB} = \mathbf{b}$ .

Let  $M$  be a point that lies  $\frac{1}{3}$  of the way along  $OA$  and  $N$  be a point that lies  $\frac{2}{3}$  of the way along  $AC$ .

Walkers have trodden a path through the grass between  $M$  and  $N$ .



- a) Find the vector  $\overrightarrow{OC}$ .

$$= \mathbf{a} + \mathbf{b}$$

[1 mark]

- b) Find the vector  $\overrightarrow{MN}$ .

$$\begin{aligned} \overrightarrow{MN} &= \frac{2}{3}\mathbf{a} + \frac{2}{3}\mathbf{b} \\ &= \frac{2}{3}(\mathbf{a} + \mathbf{b}) \end{aligned}$$

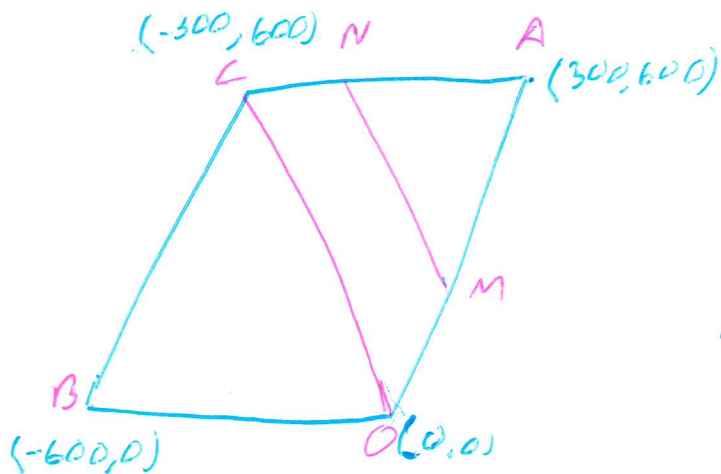
[2 marks]

- c) What can be deduced about the vectors  $\overrightarrow{OC}$  and  $\overrightarrow{MN}$  from parts (a) and (b)? And why?

$\overrightarrow{MN}$  is a multiple of  $\overrightarrow{OC}$  and  
so is parallel to  $\overrightarrow{OC}$  [1 marks]

- d) Assuming that the meadow is flat and that the with  $O$  being the origin the coordinates of  $A$ ,  $B$  and  $C$  are  $(300,600)$ ,  $(-600,0)$  and  $(-300,600)$  work out how long it takes John to walk through route  $OMNCO$  at a speed of  $0.8 \text{ ms}^{-1}$ .

[4 marks]



$$\overrightarrow{OA} = \begin{pmatrix} 300 \\ 600 \end{pmatrix} \Rightarrow OM = \begin{pmatrix} 100 \\ 200 \end{pmatrix}$$

$$\overrightarrow{NC} = \frac{1}{3} \begin{pmatrix} -600 \\ 0 \end{pmatrix} = \begin{pmatrix} -200 \\ 0 \end{pmatrix}$$

$$\overrightarrow{OC} = \begin{pmatrix} -300 \\ 600 \end{pmatrix}$$

$$\begin{aligned}\vec{MN} &= \frac{2}{3} \vec{OC} \\ &= \frac{2}{3} \begin{pmatrix} -200 \\ 400 \end{pmatrix}\end{aligned}$$

Distances:

$$|\vec{OM}| = \sqrt{50000}$$

$$|\vec{MN}| = \sqrt{200000}$$

$$|\vec{OC}| = \sqrt{450000}$$

$$|\vec{NC}| = \sqrt{40000}$$

$$\begin{aligned}\text{So total distance} &= |\vec{OM}| + |\vec{MN}| + |\vec{OC}| + |\vec{NC}| \\ &= 1541.64 + 0786\end{aligned}$$

So

$$\begin{aligned}t &= \frac{\text{distance}}{\text{speed}} \\ &= \frac{1541.64}{0.8} \\ &= 1927.050983 \text{ seconds} \\ &= 32.1 \text{ minutes}\end{aligned}$$

- 17 A ball is projected from the ground with speed  $u$  at an angle  $\alpha$  to the horizontal where the only force acting on the ball is gravity.

The  $x$ - and  $y$ -axes are horizontal and vertical, passing through the origin  $O$  in the plane of motion of the ball.

- a) Find the time of flight of the particle. ← add when it hits the ground

[3 marks]

	Horizontal motion	Vertical motion
$s$	$s_H$	0
$u$	$u \cos(\alpha)$	$u \sin(\alpha)$
$v$	$u \cos(\alpha)$	
$a$	0	$-g$
$t$	$T$	$T$

Considering horizontal motion

$$s_H = uT \cos(\alpha)$$

Considering vertical motion,

$$\begin{aligned} 0 &= u \sin(\alpha)T - \frac{1}{2}gT^2 \\ &= T(u \sin(\alpha) - \frac{1}{2}gT) \end{aligned}$$

$$\text{So } T=0 \text{ or } u \sin(\alpha) - \frac{1}{2}gT = 0$$

$$\Rightarrow u \sin(\alpha) = \frac{1}{2}gT$$

$$\Rightarrow T = \frac{2u \sin(\alpha)}{g}$$

b) Show that the equation of the path can be given by

$$y = x \tan(\alpha) - \frac{gx^2}{2u^2} (1 + \tan^2(\alpha))$$

[5 marks]

	Horizontal motion	Vertical motion
s	x	y
u	$u \cos(\alpha)$	$u \sin(\alpha)$
v	$u \cos(\alpha)$	
a	0	-g
t	t	t

Horizontal motion:

$$x = ut \cos(\alpha) \quad (1)$$

Vertical motion

$$y = u \sin(\alpha)t - \frac{1}{2}gt^2 \quad (2)$$

From (1),

$$t = \frac{x}{u \cos(\alpha)} \quad (3)$$

Substitute (3) into (2)

$$y = u \frac{x}{u \cos(\alpha)} \sin(\alpha) - \frac{1}{2}g \left( \frac{x}{u \cos(\alpha)} \right)^2$$

$$= x \tan(\alpha) - \frac{gx^2}{2u^2 \cos^2 \alpha}$$

$$= x \tan(\alpha) - \frac{gx^2}{2u^2} \sec^2(\alpha)$$

$$= x \tan(\alpha) - \frac{g x^2}{2u^2} \sec^2(\alpha)$$

$$= x \tan(\alpha) - \frac{g x^2}{2u^2} (1 + \tan^2(\alpha))$$

c) Find the range of the particle.

[3 marks]

The range is the value of  $x$  when  $t = \frac{2u \sin(\alpha)}{g}$   
from (a).

$$v_{\text{vertical}} = -u \cos(\alpha) t$$

$$x_R = u \frac{2u \sin(\alpha)}{g} \cos(\alpha)$$

$$= \frac{2u^2 \sin(\alpha) \cos(\alpha)}{g}$$

$$= \frac{u^2 \sin(2\alpha)}{g}$$

[NB: this is maximised when  $\sin(2\alpha) = 1 \Rightarrow \alpha = 45^\circ$   
when the range is  $\frac{u^2}{g}$

- 18 A block,  $A$ , of 4 kg is placed on a rough plane inclined at  $\alpha^\circ$  to the horizontal where  $\cos(\alpha) = \frac{4}{5}$ .

The coefficient of friction between the block and the plane is  $\frac{3}{5}$ .

One end of a light rope is attached to the block and passes over a smooth pulley at the top of the plane. The other end of the rope is attached to a sphere,  $B$ , of mass 7 kg which hangs vertically below the pulley. Initially  $B$  is 4m above the floor.

The system is released from rest, and as the sphere falls the block moves directly up the plane with acceleration  $a \text{ ms}^{-2}$ .

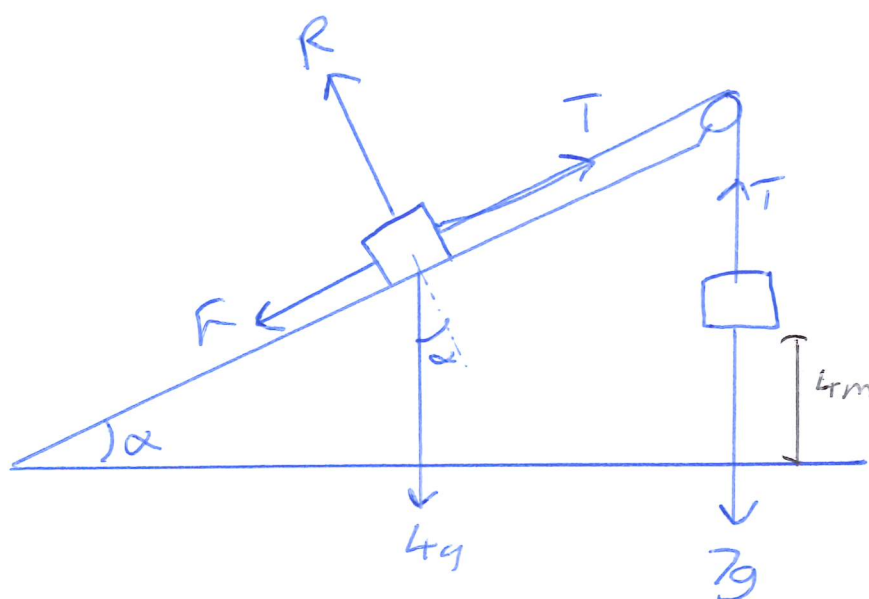
- a) What is the implication from the pulley being smooth?

[1 mark]

*Tension is the same in the rope either side of the pulley.*

- b) Draw a labelled force diagram for this situation

[3 marks]



$$\cos(\alpha) = \frac{4}{5}$$

$$\sin(\alpha) = \frac{3}{5}$$

- c) Find an equation of motion for the sphere.

$$\text{Apply } F=ma \downarrow$$

$$7g - T = 7a$$

[1 mark]

- d) Determine the tension in the rope and the acceleration of the block in terms of  $g$ .

[6 marks]

Apply  $F=ma$   $\uparrow$  // to plane

$$T - F - 4g \sin(\alpha) = 4a \quad (1)$$

Apply  $F=ma$   $\uparrow$   $\perp$  to plane

$$R - 4g \cos(\theta) = 0 \quad (2)$$

$$(2) \Rightarrow R = 4g \times \frac{4}{5} = \frac{16g}{5} \quad (3)$$

$$(1) \Rightarrow T - \frac{3}{5}R - 4g \frac{3}{5} = 4a \quad (4)$$

Sub in (3) into (4)

$$T - \frac{3}{5} \times \frac{16g}{5} - \frac{12g}{5} = 4a$$

$$\Rightarrow T - 4a = \frac{108}{25}g \quad (+)$$

$$\text{and } +T + 7a = 7g \quad (\pi)$$

$$7g - \frac{108}{25}g = 11a$$

$$\Rightarrow 11a = \frac{67}{25}g$$

$$a = \frac{67}{275}g$$

Then,

$$\begin{aligned} T &= \frac{108}{25}g + 4 \times \frac{67}{275}g \\ &= \frac{1456}{275}g \end{aligned}$$

- e) After the sphere hits the floor the string becomes slack.

Find the minimum distance between the blocks starting position and the pulley for the block to not hit the pulley.

[4 marks]

When string becomes slack there is no tension.

What's found:

$$s = 4$$

$$u = 0$$

$$v =$$

$$a = \frac{67}{275}g$$

$$t =$$

$$v^2 = u^2 + 2as$$

$$v^2 = 0 + 2 \times \frac{67}{275}g \times 4$$

$$= \frac{536}{275}g$$

When string goes slack:

Apply  $F=ma \rightarrow$

$$-l_0 g \sin \theta - F = l_0 a$$

$$l_0 a = -\frac{17g}{5} - \frac{3}{5} \times \frac{16g}{5}$$

$$= -\frac{108}{25} g$$

$$\Rightarrow a = -\frac{27}{25} g$$

Now using the new acceleration and the final speed of A whilst the rope is taut,

$$s = s$$

$$u = \sqrt{\frac{536}{275}} g$$

$$v = 0$$

$$a = -\frac{27}{25} g$$

$$t =$$

$$v^2 = u^2 + 2as$$

$$0 = \frac{536}{275} g + 2 \times \frac{-27}{25} g s$$

$$0 = \frac{536}{275} g - \frac{54}{25} g s$$

$$\Rightarrow s = \frac{\frac{536}{275}}{\frac{54}{25}}$$

$$= \frac{268}{297} \text{ m}$$

$$\approx 0.902 \text{ m.}$$

$$\text{So distance travelled is } l_0 + 0.902 \\ = 4.902 \text{ m.}$$