Dr Bennison Maths

MATHEMATICS EDUCATION EXPERT

AQA A-Level Maths 2026 Paper 3

Do not turn over the page until instructed to do so.

This assessment is out of 100 marks and you will be given 120 minutes.

When you are asked to by your teacher write your **full name** below

Name:

Total Marks: **/ 100**

Section A

1 The equation of a curve is $y = \sin(2x) + 2\cos(x)$.

Find an expression for $\dfrac{dy}{dx}$.

$\dfrac{dy}{dx} = 2\cos(2x) - 2\sin(x)$ ☑

$\dfrac{dy}{dx} = 2\cos(2x) + 2\sin(x)$ ☐

$\dfrac{dy}{dx} = -2\cos(2x) + 2\sin(x)$ ☐

$\dfrac{dy}{dx} = \cos(2x) - 2\sin(x)$ ☐

[1 mark]

2 Solve $|2x + 1| = 5$

$x = -2$ and $x = 3$ ☐

$x = -2$ and $x = -3$ ☐

$x = 2$ and $x = 3$ ☐

$x = 2$ and $x = -3$ ☑

[1 mark]

3 Which of the below is a vector of magnitude $\sqrt{32}$ in the south westerly direction?

$\begin{pmatrix} -4 \\ 4 \end{pmatrix}$ ☐

$\begin{pmatrix} -4 \\ -4 \end{pmatrix}$ ☑

$\begin{pmatrix} -6 \\ -2 \end{pmatrix}$ ☐

$\begin{pmatrix} -3 \\ -3 \end{pmatrix}$ ☐

[1 mark]

4 Find, from first principles the derivative of $y = \frac{1}{x}$.

[4 mark]

Let $f(x) = \frac{1}{x}$

Then

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{x - (x+h)}{x(x+h)}}{h}$$

$$= \lim_{h \to 0} \frac{-h}{h(x)(x+h)}$$

$$= \lim_{h \to 0} -\frac{1}{x(x+h)}$$

$$= -\frac{1}{x^2}$$

5 Solve, showing all reasoning,

$$\frac{1}{2} \times 2^{2x+4} - 2^{x+3} = -2$$

[5 marks]

$2^{2x+4} = 2^4 \times 2^{2x} = 16 \times 2^{2x}$

$2^{x+3} = 2^x \times 2^3 = 8 \times 2^x$

Hence,

$\frac{1}{2} \times 2^{2x+4} - 2^{x+3} = -2$

$\Rightarrow \frac{1}{2} \times 16 \times 2^{2x} - 8 \times 2^x = -2$

$\Rightarrow 8 \times 2^{2x} - 8 \times 2^x + 2 = 0$

Let $y = 2^x$, then

$8y^2 - 8y + 2 = 0$

$(4y - 2)(2y - 1) = 0$

So $4y - 2 = 0$ or $2y - 1 = 0$

$\Rightarrow y = \frac{1}{2}$ or $y = \frac{1}{2}$

but $y = 2^x$, so

$2^x = \frac{1}{2} \Rightarrow x = -1$ as the only real solution

6 Consider the function $f(x) = 3x\sin(x)$.

a) Show that the x-coordinates of the inflection points of the curve

$$f(x) = 3x\sin(x)$$

satisfy the equation.

$$x = 2\cot(x)$$

[4 marks]

$f(x) = 3x\sin(x)$

$f'(x) = 3\sin(x) + 3x\cos(x)$

$f''(x) = 3\cos(x) + 3x \times -\sin(x) + 3\cos(x)$

$= 6\cos(x) - 3x\sin(x)$

At an inflection point

$$\frac{d^2y}{dx^2} = 0$$

So

$6\cos(x) - 3x\sin(x) = 0$

$\Rightarrow$ $6\cos(x) = 3x\sin(x)$

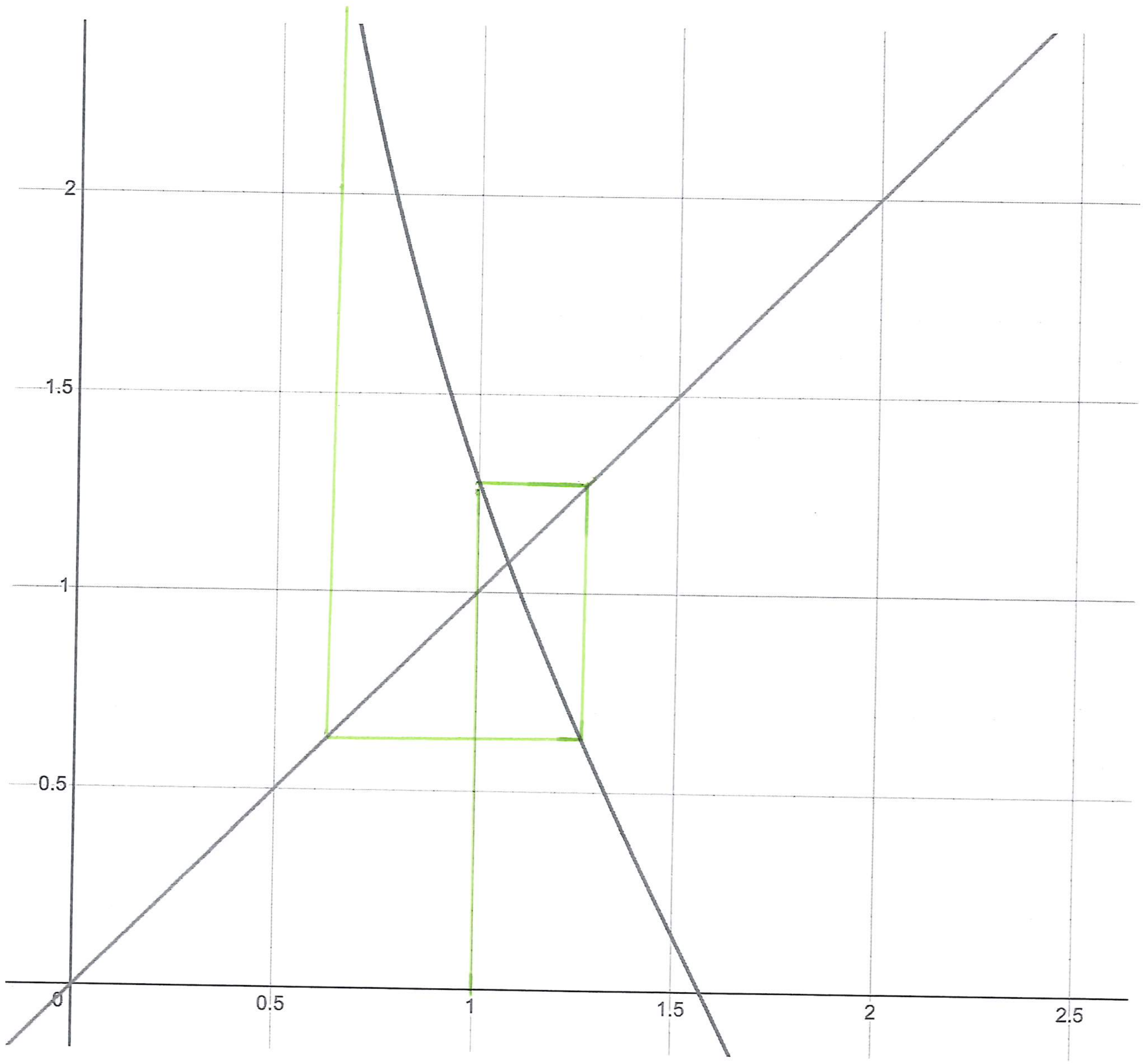
$x = 2\cot(x)$

b) Jess tries to find an inflection point of the curve by using the iteration

$$x_{n+1} = 2\cot(x_n)$$

with $x_0 = 1$.

With reference to annotations you make on the diagram below explain how you know that this iteration will not converge to an inflection point.



[3 marks]

This is a diverging cobweb diagram and so the iteration doesn't converge

7 Let $p(x) = 3x^4 - 4x^3 + ax^2 + bx + 17$.

Given that,

- $p(3) = -388$
- $(3, -388)$ is a stationary point on the curve.

Prove that the point with x-coordinate, $x = 3$, is the only stationary point of $p(x)$.

[7 marks]

$p(3) = -388$

$\Rightarrow 3 \times 3^4 - 4 \times 3^3 + 9a + 3b + 17 = -388$

$9a + 3b = -540$ (1)

$(3, -388)$ a stationary point means $p'(x) = 0$.

$p'(x) = 12x^3 - 12x^2 + 2ax + b$

So $p'(3) = 0$

$\Rightarrow 324 - 108 + 6a + b = 0$

$6a + b = -216$ (2)

Solving (1) and (2)

$a = -12$

$b = -144$

Hence,

$p(x) = 3x^4 - 4x^3 - 12x^2 - 144x + 17$

Stationary points are when $\frac{dy}{dx} = 0$, i.e. $p'(x) = 0$

So $p'(x) = 0$

$$\Rightarrow \quad 12x^3 - 12x^2 - 24x - 144 = 0$$

$$\Rightarrow \quad (x-3)(12x^2 + 24x + 48) = 0$$

$$\Rightarrow 12(x-3)(x^2 + 2x + 4) = 0$$

Consider the discriminant of x^2+2x+4.

$$\Delta = 2^2 - 4 \times 4 \times 1$$
$$= 4 - 16$$
$$= -12$$

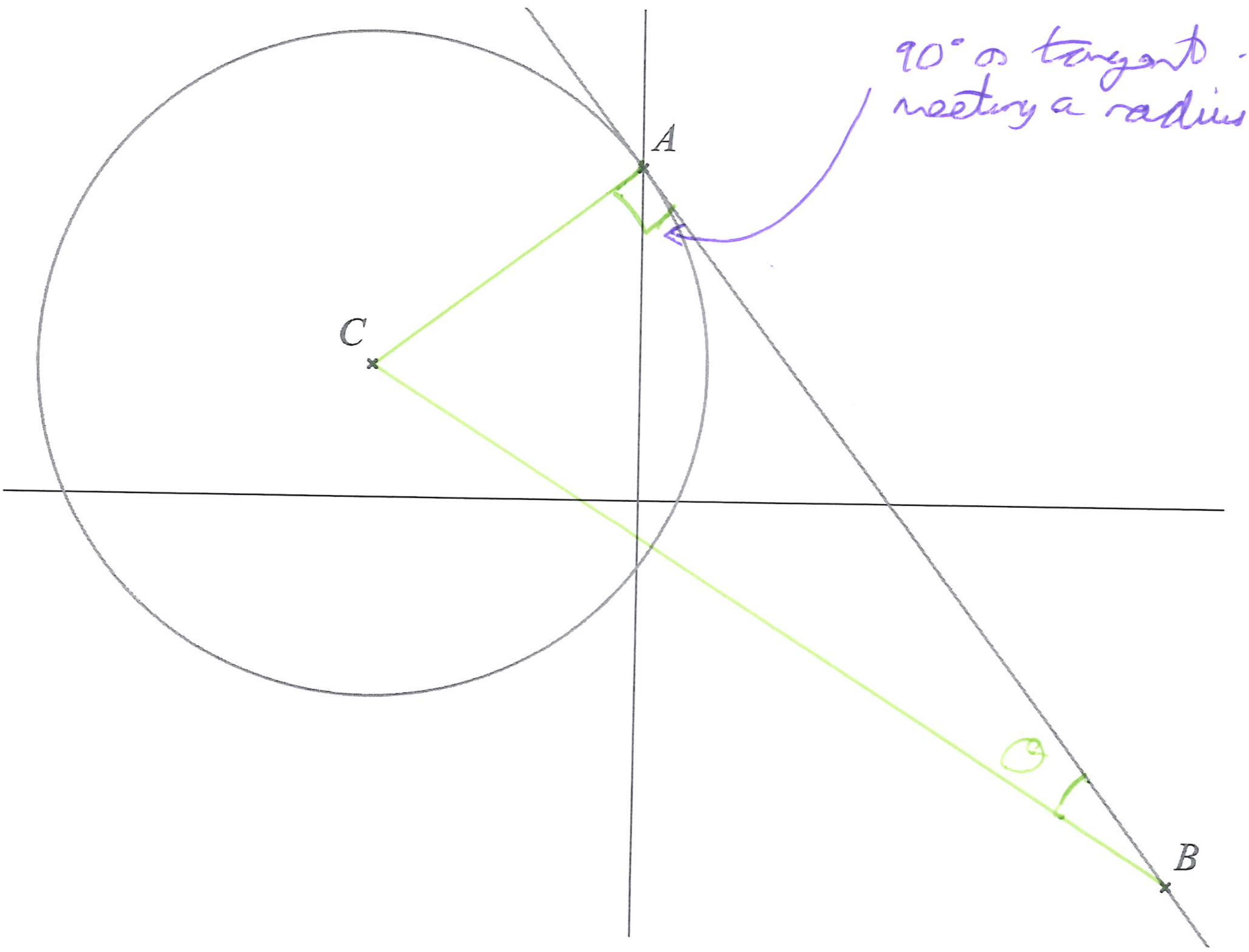
So the quadratic x^2+2x+4 has no real roots.
Hence, the derivative $p'(x) = 0$ has $x=3$ as the only real root.

8 The diagram below shows a circle with equation

$$x^2 + 8x + y^2 - 4y - 5 = 0$$

which has centre C and intersects the $y-$axis at the point A.

The straight line is a tangent to the circle at A and passes through the point $B\left(8, -\dfrac{17}{3}\right)$.



Find the angle ABC, in degrees to two decimal places.

[7 marks]

$$x^2+8x+y^2-4y-5=0$$

$$(x+4)^2-16+(y-2)^2-4-5=0$$

$$(x+4)^2+(y-2)^2=25$$

Centre $(-4,2)$.

When $x=0$,

$$4^2+(y-2)^2=25$$
$$(y-2)^2=9$$
$$y-2=\pm 3$$
$$y=5 \text{ or } -1$$

Hence $A(0,5)$

$$|AB| = \sqrt{(8-0)^2+\left(-\frac{17}{3}-5\right)^2}$$

$$= \sqrt{8^2+\left(\frac{-32}{3}\right)^2}$$

$$= \sqrt{\frac{1600}{9}}$$

$$= \frac{40}{3}$$

Hence $\tan(\theta) = \left(\frac{5}{\frac{40}{3}}\right)$

$$\theta = \arctan\left(\frac{15}{40}\right) = 20.55604522$$
$$= 20.6°$$

9 Using a suitable substitution find the exact value of

$$I = \int_0^4 \frac{2}{2+\sqrt{1+2x}}\,\mathrm{d}x$$

[6 marks]

Let $u = 2 + \sqrt{1+2x} = 2 + (1+2x)^{1/2}$

$$\frac{du}{dx} = \frac{1}{2}(1+2x)^{-1/2} \times 2$$

$$= \frac{1}{\sqrt{1+2x}}$$

$$= \frac{1}{u-2}$$

So $dx = (u-2)\,du$

When $x = 0$, $u = 3$

$x = 4$, $u = 5$

Hence,

$$I = \int_0^4 \frac{2}{2+\sqrt{1+2x}}\,dx$$

$$= \int_3^5 \frac{2}{u}(u-2)\,du$$

$$= 2\int_3^5 \frac{u-2}{u}\,du$$

$$= 2\int_3^5 1 - \frac{2}{u}\,du$$

$$= 2\left[u - 2\ln|u|\right]_3^5$$

$$= 2\left[\left(5 - 2\ln(5) - \left(3 - 2\ln(3)\right)\right)\right]$$

$$= 2\left[3 + 2\ln(3) - 2\ln(5)\right]$$

$$= 6 + 4\ln\left(\frac{3}{5}\right)$$

10 A curve C is defined by the parametric equations

$$x = 3\cos(t),\ y = 4\sin(2t)\ ,\ 0 \leq t \leq 2\pi$$

a) Find the equation of the normal to the curve at the point where $t = \dfrac{\pi}{6}$ in the form $16y - 2bx = a\sqrt{b}$ where a and b are integers to be found.

[4 marks]

$$\frac{dx}{dt} = -3\sin(t)$$

$$\frac{dy}{dt} = 8\cos(2t)$$

So $$\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt}$$

$$= \frac{8\cos(2t)}{-3\sin(t)}$$

When $t = \frac{\pi}{6}$

$$x = 3\cos\left(\frac{\pi}{6}\right) = \frac{3\sqrt{3}}{2}$$

$$y = 4\sin\left(\frac{\pi}{3}\right) = 2\sqrt{3}$$

$$\frac{dy}{dx} = \frac{8\cos(\pi/3)}{-3\sin(\pi/6)} = -\frac{4}{3/2} = \frac{-8}{3}$$

So gradient of the normal is $\frac{3}{8}$

Show that the shaded area is p square units where p is a prime number to be found.

[6 marks]

B is when the normal crosses the x-axis, ie when $y = 0$

$$-6x = 23\sqrt{3}$$
$$= -\frac{23}{6}\sqrt{3}$$

Point D has x-coordinate is $\frac{3\sqrt{3}}{2}$

Area of triangle $= \frac{1}{2} \times \left(\frac{23}{6}\sqrt{3} + \frac{3\sqrt{3}}{2}\right) \times 2\sqrt{3}$

$$= \frac{1}{2} \times \frac{16}{3}\sqrt{3} \times 2\sqrt{3}$$

$$= 16$$

Area of the other bit $= \int y \frac{dx}{dt}\, dt$

$$= \int_0^{\pi/6} 4\sin(2t)\,{}^{-3}\sin(t)\, dt$$

$$= -12\int_0^{\pi/6} \sin(2t)\sin(t)\, dt$$

$$= -24\int_0^{\pi/6} \sin^2(t)\cos(t)$$

$$= \left[-24\, \frac{1}{3}\sin^3(t)\, dt\right]_0^{\pi/6}$$

$$= -1 \quad \text{so} \quad \text{area} = 1$$

Hence,

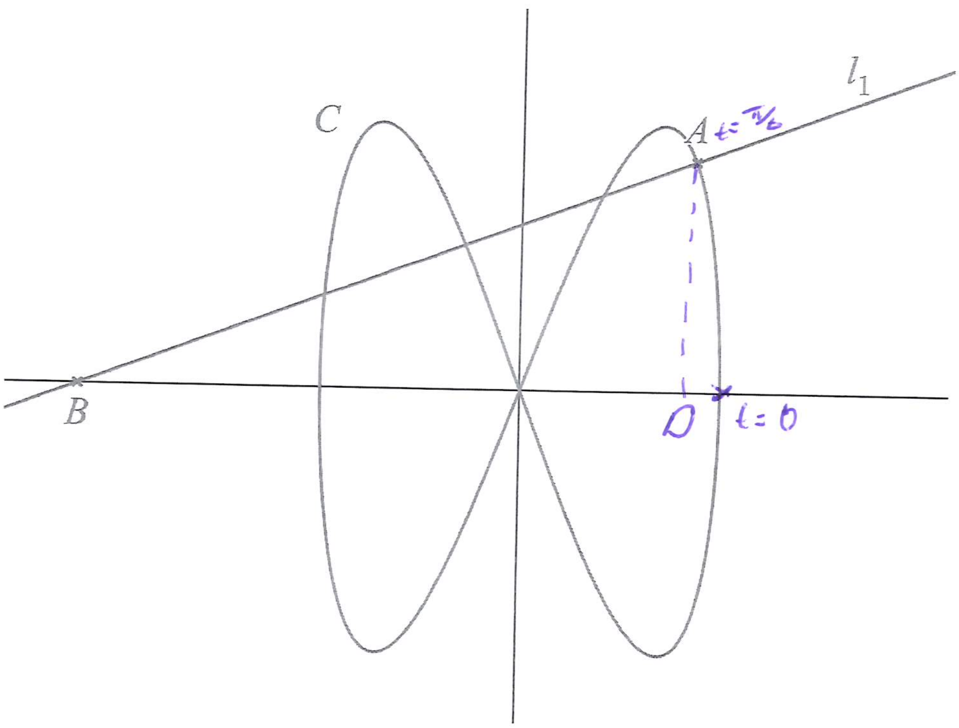
$$y - 2\sqrt{3} = \frac{3}{8}\left(x - \frac{3\sqrt{3}}{2}\right)$$

$$y = \frac{3}{8}x + \frac{23\sqrt{3}}{16}$$

$$\Rightarrow 16y = 6x + 23\sqrt{3}$$

$$\Rightarrow 16y - 6x = 23\sqrt{3}$$

b) The image below shows a sketch of the curve C and the normal found in (a). This normal intersects the $x-$axis at the point B.



Hence area of shaded region is

$16 + 1 = 17$ which is prime.

Section B

11 Let $X \sim \mathrm{N}\left(25, 2.5^2\right)$

The values a and b are such that $\mathrm{P}\left(a \leq x \leq b\right) \approx 0.68$.

Identify the correct pair of values.

- [x] $a = 22.5$ and $b = 27.5$
- [ ] $a = 22.5$ and $b = 25$
- [ ] $a = 20$ and $b = 30$
- [ ] $a = 25$ and $b = 30$

[1 mark]

12 The discrete random variable Y is modelled as a binomial distribution $Y \sim B(n, p)$.

Given that,

- The mean of Y is 4
- The variance of Y is 3.2

What is the correct pairs of values?

- [ ] $n = 20$ and $p = 0.8$
- [ ] $n = 40$ and $p = 0.4$
- [x] $n = 20$ and $p = 0.2$
- [ ] $n = 40$ and $p = 0.4$

[1 mark]

13 Emily is wishing to investigate there political affiliation of the 3300 households in her local area.

She decides to obtain a sample of 150 households.

She enlists her friend, Lily, who suggests they select every 22nd household in a list of all the households until a sample of size 150 has been collected.

a) What is the population for this study.

The 150 households selected ☐

All 3300 households ☑

Lily and Emily ☐

Houses that vote conservative. ☐

[1 mark]

b) What is the sampling method suggested by Lily.

Systematic ☑

Opportunity ☐

Quota ☐

Stratified ☐

[1 mark]

14 The mass of a Malling Centenary strawberry is modelled by a normal distribution $M \sim \mathrm{N}\left(\mu, \sigma^2\right)$.

a) Given that

$$\mathrm{P}(M \leq 19) = 0.1586$$

and that

$$\mathrm{P}\left(N > 25\right) = 0.0228$$

find μ and σ.

[6 marks]

$P(M \leq 19) = 0.1586$

$\Rightarrow \dfrac{19-\mu}{\sigma} = -1$

$\Rightarrow \mu - \sigma = 19$ (1)

$P(N > 25) = 0.0228$

$\Rightarrow \dfrac{25-\mu}{\sigma} = 1.999$

$\Rightarrow \mu + 1.999\sigma = 25$ (2)

Solving (1) and (2)

$\mu = 21$

$\sigma = 2$

b) Leaf Farm tries a new growing regime and a sample of 20 berries is taken which weighs a total of 445 grams.

Is there evidence that the regime change has made a difference to the average weight.

Perform a hypothesis test to investigate.

[6 marks]

From ~~the~~ sample $\bar{x} = \frac{445}{20} = 22.25$

Let $M \sim N(21, 2^2)$, then $\bar{M} \sim N\left(21, \frac{2^2}{20}\right)$

$H_0 : \mu_0 = 21$

$H_1 : \mu_1 \neq 21$

Using a 5% significance level, ~~two~~ tailed test so 2.5% in each tail

$P(\bar{M} \geq 22.25) = 0.002594$

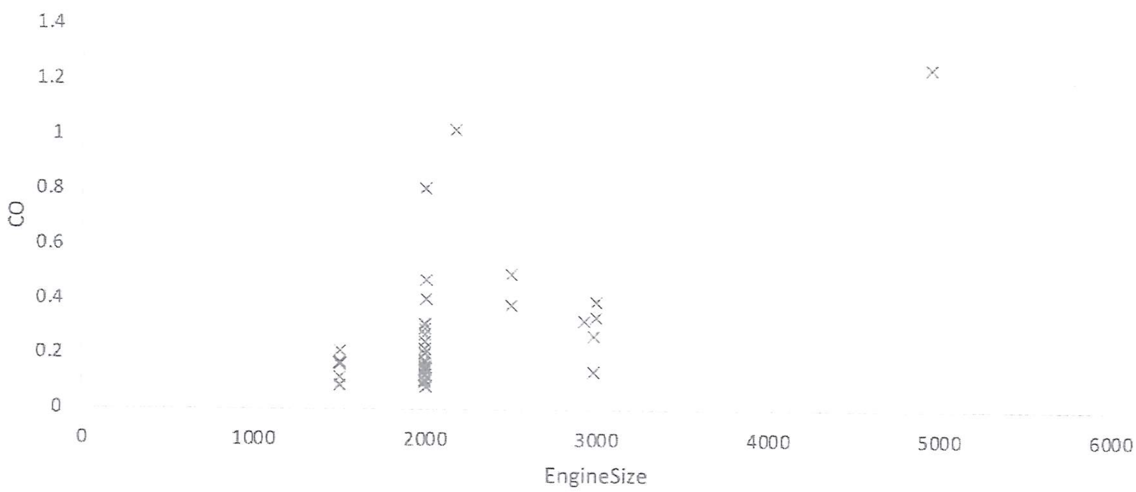
$0.002594 < 0.025$

So there is sufficient evidence to reject H_0.

Hence, sufficient evidence to conclude that the regime change has made a difference to the average weight.

15 Charlie is using the Large Data Set to explore whether there is a correlation between engine size and CO emissions.

He produces the below scatter graph for a sample of 50 cars from a single manufacturer.



a) What kind of correlation is shown in the scatter graph above?

[1 mark]

Week positive

b) Using your knowledge of the Large Data Set explain the presence of the vertical lines in the graph.

[1 mark]

Engine sizes are often in fixed values (CC), ie unlikely to have a 2775 CC engine.

c) Charlie calculates the correlation coefficient between the engine size and CO emissions for his sample. He obtains a value of $r = 0.62$.

He therefore believes there is a positive correlation between engine size and CO emissions.

Given that for $n = 50$ the critical value for a one-tailed test at the $1\,\%$ level is 0.3281, carry out a hypothesis test at the $1\,\%$ level of significance to determine if the sample provides evidence to support Charlie's belief.

[4 marks]

$H_0: \rho = 0$

$H_1: \rho > 0$

$0.62 > 0.3281$

So, sufficient evidence to reject H_0.
Sufficient evidence to conclude that there is a positive correlation between engine size and CO emissions

d) Charlie's friend Elif says that using data from the large data set she can conclude that, for all cars in the UK, the engine size can be used to predict the CO emissions.

Is she correct?

[2 marks]

No

The large data set doesn't contain all cars or all regions

16 Jars of Little Scarlet jam (James Bond's jam of choice) are sold to retailers in trays of 48 jars.

It is known that $5\ \%$ of jars have a blurred expiry date.

a) Let X be the random variable "Number of jars in a tray of 48 that have a blurred expiry label".

State an assumption, in context, necessary for the binomial distribution to be used to model the random variable X.

[1 mark]

The probability of a label having a blurred expiry date is constant.

b) Find the probability that there is at most 2 jars with a blurred expiry date in a given tray.

[2 marks]

Let X be the R.V = Number of jars in a tray of 48 with blurred expiry. Then

$X \sim B(48, 0.05)$

$P(X \leq 2) = 0.56704563 54$

$= 0.5670$ to 4dp

c) Given that Fine Provisions buys 8 trays. What is the probability that there are exactly 3 trays with at most 2 jars with blurred labels?

[2 marks]

Let Y be the drv = Number of trays in 8 with at most 2 jars with blurred expiry date. Then $Y \sim B(8, 0.5670)$

$P(X = 3) = \cancel{0.2287986843}\ 0.1553290655$

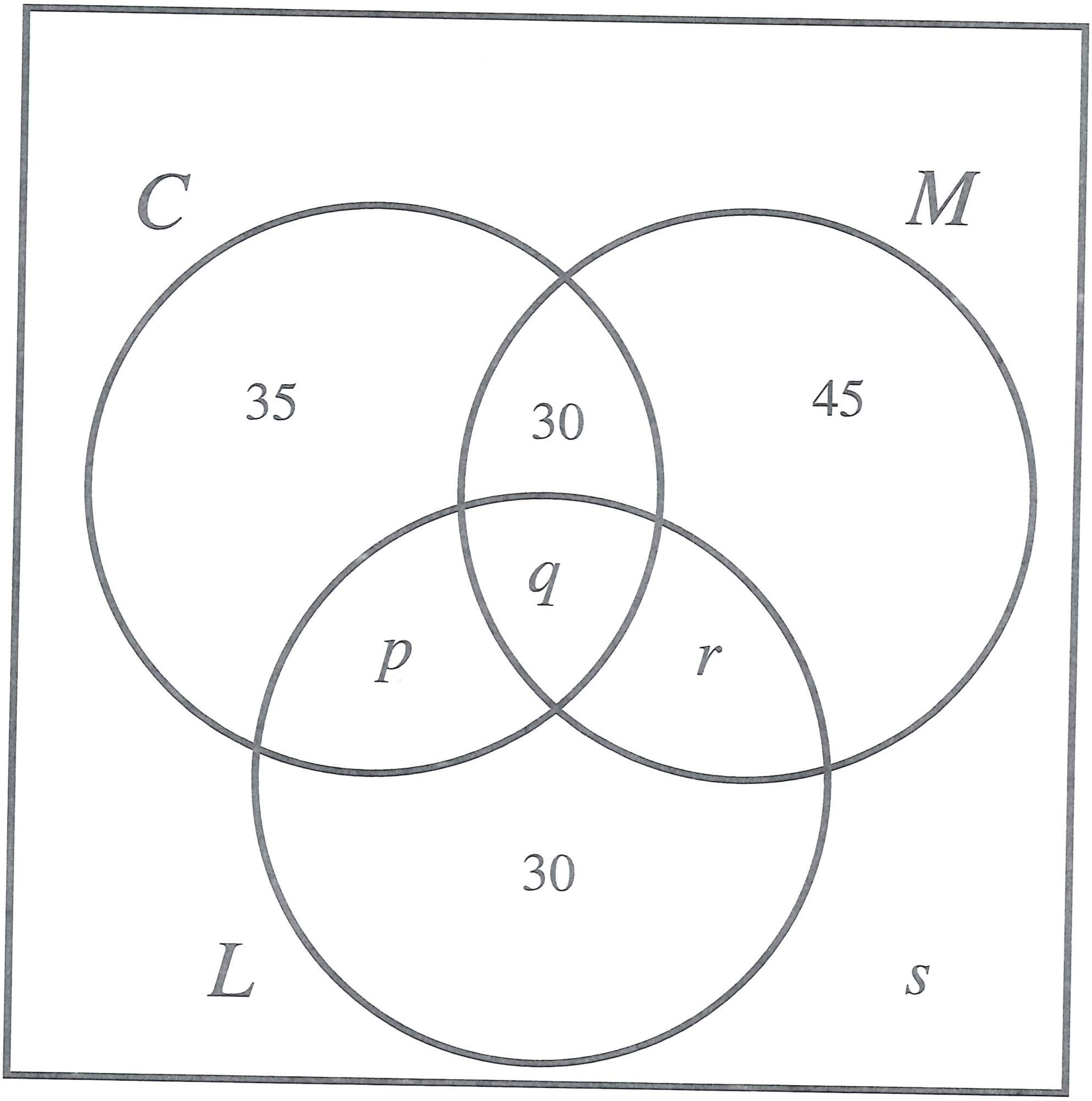
$= \cancel{0.2288}\ 0.1553$

17 In a sixth form of 220 students they are all asked whether they like to listen to three country artists.

Define the events C, M and L as follows:

- C : they like listening to The Castellows
- M : they like listening to Megan Moroney
- L : they like listening to Luke Combs

A Venn diagram is constructed showing their listening preferences as below, where p, q, r and s are whole numbers.



a) Given that 55 students like listening to both The Castellows and Megan Moroney and that $\mathrm{P}(C) = \frac{1}{2}$,

(i) Find the value of q

[1 mark]

$55 - 30 = 25$

(ii) Find the value of p.

[2 marks]

$\frac{35+30+25+p}{220} = \frac{1}{2} \Rightarrow p = 20$

b) Given that $\mathrm{P}(L \mid M) = \frac{8}{23}$, find r.

[3 marks]

$\frac{q+r}{30+45+q+r} = \frac{8}{23}$

So

$23(25+r) = 8(100+r)$

$575 + 23r = 800 + 8r$

$\Rightarrow 15r = 225$

$r = 15$

c) What is the value of s?

[1 mark]

$220 - (35 + 30 + 45 + 20 + 25 + 15 + 30)$

$= 20$

d) Determine, showing all reasoning, whether the events C and $M \cap L$ are independent.

[3 marks]

$P(C \cap (M \cap L)) = \frac{25}{220} = \frac{5}{44}$

$P(G) = \frac{110}{220}$

$P(M \cap L) = \frac{40}{220}$

Then $P(G) \times P(M \cap L) = \frac{110}{220} \times \frac{40}{220} = \frac{1}{11}$

$\frac{1}{11} \neq \frac{5}{44}$ so C and $M \cap L$ are not independent ~~[illegible]~~ events

18 The discrete random variable X takes integers which are multiples of 3 from 3 to $3n$ inclusive.

The probability distribution of X is given by

$$P(X = x) = \frac{x}{k}, \qquad x = 3,6,9,\ldots,3n$$

a) Show that $k = \dfrac{3n(n+1)}{2}$

[4 marks]

Probabilities sum to 1,

so

$$\frac{3}{k} + \frac{6}{k} + \frac{9}{k} + \cdots + \frac{3n}{k} = 1$$

$$\Rightarrow \frac{3}{k}(1 + 2 + 3 + \cdots + n) = 1$$

arithmetic series
$a = 1,\ d = 1$

$$\Rightarrow \cancel{\frac{3}{k}\frac{n}{2}[2a + (n-1)d]}$$

$$\Rightarrow \frac{3}{k}\frac{n}{2}(2\times 1 + (n-1)\times 1] = 1$$

So $$\frac{3n(n+1)}{2k} = 1$$

$$\Rightarrow \quad k = \frac{3n(n+1)}{2}$$

b) When $n = 21$ find the exact value of $P(15 < X \leq 28)$

[3 marks]

For $n = 21$,

$$k = \frac{3 \times 21 \times 22}{2}$$

$$= 693$$

For $15 < X \leq 28$ consider $18, 21, 24, 27$

So

$$P(15 < X \leq 28) = \frac{18}{k} + \frac{21}{k} + \frac{24}{k} + \frac{27}{k}$$

$$= \frac{1}{693}(18 + 21 + 24 + 27)$$

$$= \frac{10}{77}$$

c) When $n = 21$, a random value, h, of X is taken and the below quadratic equation in y is formed

$$y^2 + hy + 4h = 7$$

Find the exact probability that the equation has no real roots.

[5 marks]

Consider $y^2+hy+4h=7$

$\Rightarrow y^2+hy+(4h-7)=0 \quad (\dagger)$

For no real roots we need the discriminant to be less than zero.

$b^2-4ac<0$

$\Rightarrow \quad h^2-4\times1\times(4h-7)<0$

$\Rightarrow \quad h^2-16h+28<0$

$\Rightarrow \quad (h-2)(h-14)<0$

So no real roots for

$2<h<14$

$\Rightarrow h=3,6,9,12$

So

$P(\text{equation has no real roots}) = \frac{1}{693}(3+6+9+12)$

$= \frac{10}{231}$