Dr Bennison Maths

MATHEMATICS EDUCATION EXPERT

Edexcel A-Level Maths 2026 Paper 2

Do not turn over the page until instructed to do so.

This assessment is out of 100 marks and you will be given 120 minutes.

This is **not a predicted paper** but is an opportunity to practice some topics likely to come up.

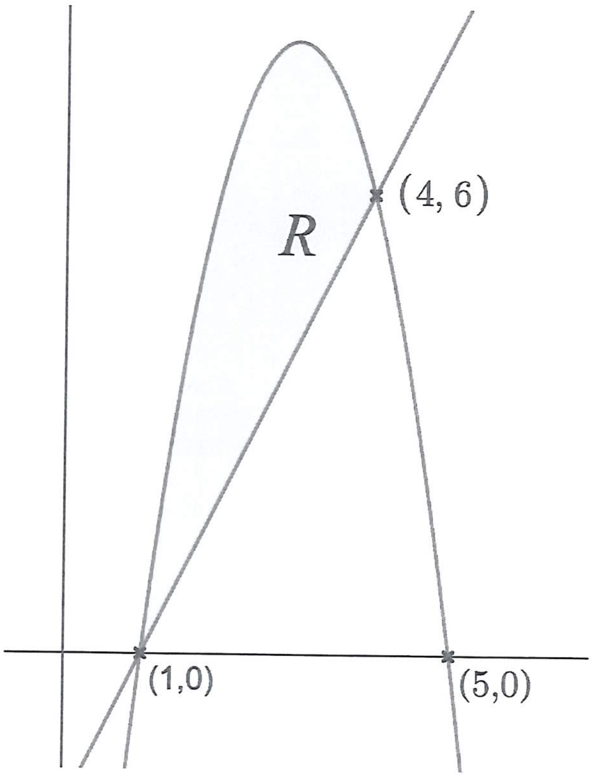
When you are asked to by your teacher write your **full name** below

Name:

Total Marks: **/ 100**

1

The figure below shows a sketch of a curve C and a straight line l.



Given that

- C has an equation $y = f(x)$ where $f(x)$ is a quadratic expression in x.
- C cuts the x−axis at 1 and 5.
- l cuts the x−axis at $(1,0)$ and intersects the curve at the point $(4,6)$.

Use inequalities to define the region R.

(5)

Line l:

$$m = \frac{6-0}{4-1} = 2$$

$$y - 0 = 2(x - 1)$$

$$\Rightarrow \quad y = 2x - 2$$

Quadratic:

$y = a(x-1)(x-5)$ passing through $(4, 6)$ so

$6 = a(4-1)(4-5)$
$= a\,{}^{-}3$

$\Rightarrow a = -2$
and the quadratic is $y = -2(x-1)(x-5)$

So, all ~~pots~~ points in R satisfy

$2x - 2 \leq y \leq -2(x-1)(x-5)$

2

Given that θ is measured in radians, prove, from first principles, that the derivative of $\sin(\theta)$ is $\cos(\theta)$.

You may assume that as $h \to 0$, $\dfrac{\sin(h)}{h} \to 1$ and that $\dfrac{\cos(h)-1}{h} \to 0$

(5)

Let $f(x) = \sin(x)$.

Then, by definition,

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sin(x+h) - \sin(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sin(x)\cos(h) + \cos(x)\sin(h) - \sin(x)}{h}$$

$$= \lim_{h\to 0} \left[\sin(x)\left(\frac{\cos(h)-1}{h}\right) + \cos(x)\left(\frac{\sin(h)}{h}\right) \right]$$

$\to 0$ as $h \to 0$ $\qquad$ $\to 1$ as $h \to 0$

$$= \cos(x)$$

3

a) Given that both $(x+1)$ and $(x-3)$ are both factors of

$$p(x) = x^4 + 5x^3 + ax^2 + bx - 36,$$

find the values of a and b.

(4)

b) Hence, fully factorise $p(x)$ as a product of linear factors.

(3)

c) Sketch $p(x)$.

(2)

a) By the factor theorem, $(x+1)$ and $(x-3)$ being factors means $p(-1)=0$ and $p(3)=0$ respectively

$p(-1)=0 \Rightarrow 1 + -5 + a - b - 36 = 0$

$\Rightarrow a - b = 40$ ①

$p(3)=0 \Rightarrow 81 + 135 + 9a + 3b - 36 = 0$

$\Rightarrow 9a + 3b = -180$ ②

Solving ① and ②

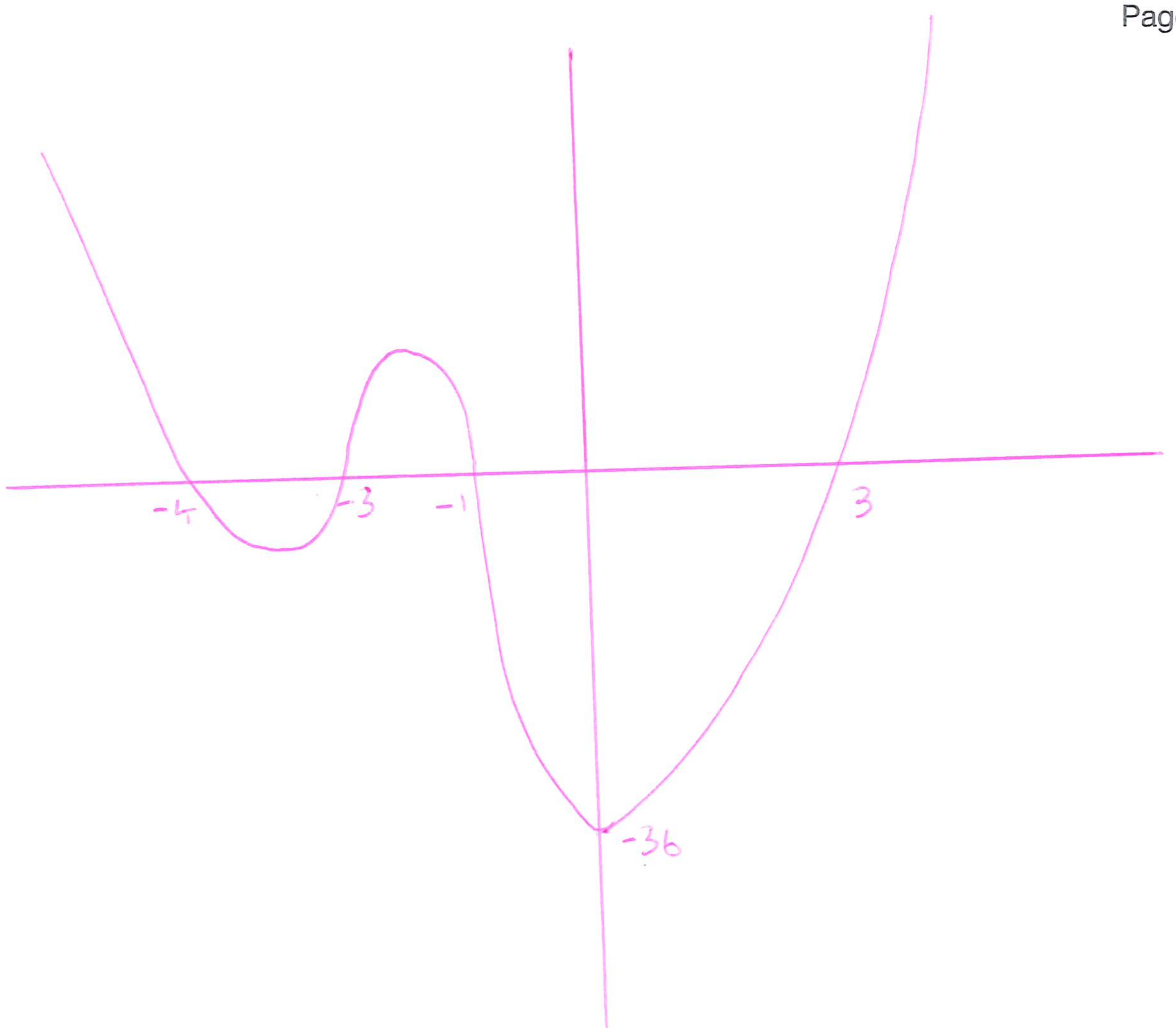
$a = -5$

$b = -45$

Hence,
$p(x) = x^4 + 5x^3 - 5x^2 - 45x - 36$

b) $x^4 + 5x^3 - 5x^2 - 45x - 36 = (x^2 - 2x - 3)(x^2 + 7x + 12)$

$= (x+1)(x-3)(x+3)(x+4)$


-4
-3
-1
3
-36

4

Jess is modelling the height, h, of a daffodil at the time t, in weeks, after the shoot appeared.

She models it with the function

$$h^2 = at + b, \qquad 0 \leq t \leq 6$$

Given that

- the height of the daffodil was 15 cm at $t = 2$.
- the height of the daffodil was 18 cm at $t = 3$.

a) Find a complete equation for the model, giving the values of a and b to three significant figures.

(4)

(b) Evaluate this model's suitability as a model for the whole of the time period.

(2)

$15^2 = 2a + b$ ①

$\Rightarrow 225 = 2a + b$

and $18^2 = 3a + b$

$\Rightarrow 324 = 3a + b$ ②

Solving ① and ②, $a = 99$, $b = 27$

Hence $h^2 = 99t + 27$

$= 99.0t + 27.0$ to 3 sig fig

b) When $t = 0$,

$$h^2 = 27$$

$$h \approx 5.2\text{cm}$$

This is unrealistic as a new shoot will not be instantly $\approx$ 5cm.

5 In a secluded woodland some rare fungus grows on an oak stump.

The area, A cm², of the fungus is measured for a few days and is shown to follow the model

$$A = 3 + 5e^{0.2t}$$

where t is the number of days after the fungus was first observed.

a) Sketch a graph of A against t, showing any intersections with the coordinate axes.

[1 mark]

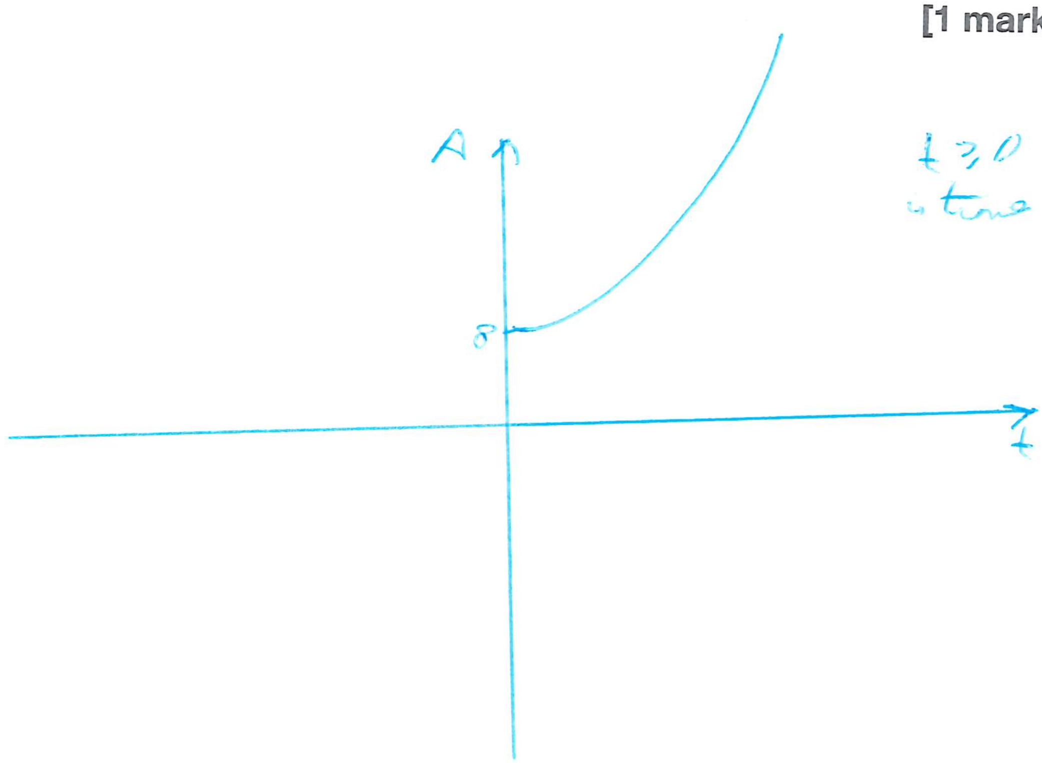
b) Find the number of days for the area of the fungus to triple in size.

[3 marks]

c) Critique the model.

[1 mark]

a)



$t \geq 0$ as it is time

b) When $t = 0$, $A = 3 + 5e^0$

$= 8$

So triple in size means $A = 24$

Hence,

$$24 = 3 + 5e^{0.2t}$$

$$\Rightarrow \quad 5e^{0.2t} = 21$$

$$e^{0.2t} = \frac{21}{5}$$

$$\ln(e^{0.2t}) = \ln\left(\frac{21}{5}\right)$$

$$0.2t\ln(e) = \ln\left(\frac{21}{5}\right)$$

$$\Rightarrow \quad 0.2t = \ln\left(\frac{21}{5}\right)$$

$$t = 5\ln\left(\frac{21}{5}\right)$$

$$= 7.1754226$$

So the area of fungus triples in size in approximately 7.2 days.

6 Solve,

a) Solve $2\log_2(x+3) - \log_2(2x) = 3$ **(2)**

b) Use the substitution $y = 2^{3x}$ to solve

$$64^x - 2^{3x+2} - 5 = 0$$

(3)

$$2\log_2(x+3) - \log_2(2x) = 3$$

$$\Rightarrow \quad \log_2\left((x+3)^2\right) - \log_2(2x) = 3$$

$$\Rightarrow \quad \log_2\left(\frac{(x+3)^2}{2x}\right) = 3$$

$$\frac{(x+3)^2}{2x} = 8$$

$$x^2+6x+9 = 16x$$

$$x^2 - 10x + 9 = 0$$

$$(x-1)(x-9) = 0$$

$$\Rightarrow \quad x = 1 \text{ and } x = 9$$

b) Let $y = 2^{3x}$

Then $y^2 = (2^{3x})^2 = 2^{6x} = (2^6)^x = 64^x$

and $2^{3x+2} = 2^{3x} \times 2^2 = 4y$

Hence,

$$64^x - 2^{3x+2} - 5 = 0$$

$$\Rightarrow y^2 - 4y - 5 = 0 \quad \text{if } y = 2^{3x}$$

Hence,

$$(y-5)(y+1) = 0$$

so, $y = 5$ or $y = -1$

As $y = 2^{3x}$, $y = -1$ gives no solutions.

Hence,

$$y = 5$$
$$2^{3x} = 5$$

so

$$\log_2(2^{3x}) = \log_2(5)$$
$$3x \log_2(2) = \log_2(5)$$
$$3x = \log_2(5)$$
$$x = \frac{1}{3}\log_2(5)$$

7

The graph of $y = \dfrac{20}{1+\sqrt{x}}$ is shown below.

a) Use the Trapezium Rule with 7 ordinates to find an approximation to the shaded area.

(4)

b) Use your answer to part (a) to state an approximate value of

$$\int_2^8 \frac{10}{1+\sqrt{x}}\, dx$$

(1)

c) By finding analytically $\displaystyle\int_2^8 \frac{10}{1+\sqrt{x}}\, dx$, find the percentage error made in the approximation performed in parts (a) and (b).

(7)

(d) Use part (a) to find $\displaystyle\int_6^{12} \frac{20}{1+\sqrt{x+4}}$

(2)

a) 7 ordinates ⇒ 6 strips

So $h = \frac{8-2}{6} = 1$

x_i	y_i
2	8.284271247
3	7.320508076
4	6.6666666 (20/3)
5	6.180339887
6	5.797958971
7	5.485837704
8	5.224077499

Hence,

$$\text{area} = \tfrac{1}{2} \times 1 \times \{8.2843 + 5.2240 + 2(7.321 + 6.6667 + 6.180 + 5.798 + 5.486)\}$$

$$= 38.2054568$$

$$= 38.205 \text{ to 3dp}$$

b) Hence,

$$\int_2^8 \frac{10}{1+\sqrt{x}}\,dx = \frac{1}{2}\int_2^8 \frac{20}{1+\sqrt{x}}\,dx$$

$$= 19.10274284$$

$$= 19.103 \text{ to } 3\text{dp}$$

c)

$$I = \int_2^8 \frac{10}{1+\sqrt{x}}\,dx$$

Let $u = \sqrt{x} \Rightarrow \frac{du}{dx} = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

$$\Rightarrow dx = 2\sqrt{x}\,du$$

$$= 2u\,du$$

when $x = 2,\ u = \sqrt{2}$

$x = 8,\ u = \sqrt{8}$

then,

$$I = 10\int_{\sqrt{2}}^{\sqrt{8}} \frac{1}{1+u}\cdot 2u\,du$$

$$= 20\int_{\sqrt{2}}^{\sqrt{8}} \frac{1}{1+u}\,du$$

$$= 20\int_{\sqrt{2}}^{\sqrt{8}} 1 - \frac{1}{1+u}\,du$$

$$= 20\left[u - \ln(1+u)\right]_{\sqrt{2}}^{\sqrt{8}}$$

$$= 20\left[\sqrt{8} - \ln(1+\sqrt{8}) - (\sqrt{2} - \ln(1+\sqrt{2}))\right]$$

$$= 19.06266206$$

So

$$\% \text{ error} = \left| \frac{19.06266206 - 19.10274284}{19.06266206} \right| \times 100$$

$$= 0.21\%$$

d) Consider

$$I_2 = \int_6^{12} \frac{20}{1 + \sqrt{x-4}} \, dx$$

Let $u = x - 4$ then $\frac{du}{dx} = 1 \Rightarrow du = dx$

When $x = 6$, $u = 2$

$x = 12$, $u = 8$

Hence

$$I_2 = \int_2^8 \frac{20}{1 + \sqrt{u}} \, du$$

which is the integral approximated in (a).

Hence,

$$I_2 \approx 38.205$$

8

Geoff bought a classic car in 1996 for £9800.

A classic car specialist valued the car at 5 year intervals as shown in the table below.

Year	1996	2001	2006	2011
Value	9800	19500	39100	79000

The valuer suggests that the valuation price can be modelled by the equation $V = a \times b^t$, where t is the number of years after 1996.

a) Find the linearised form of the model given above.

(2)

b) Complete the table below:

t	0	5	10	15
$\log_{10}(V)$				

(2)

c) By plotting a graph on the axes below, estimate the values of a and b.

(4)

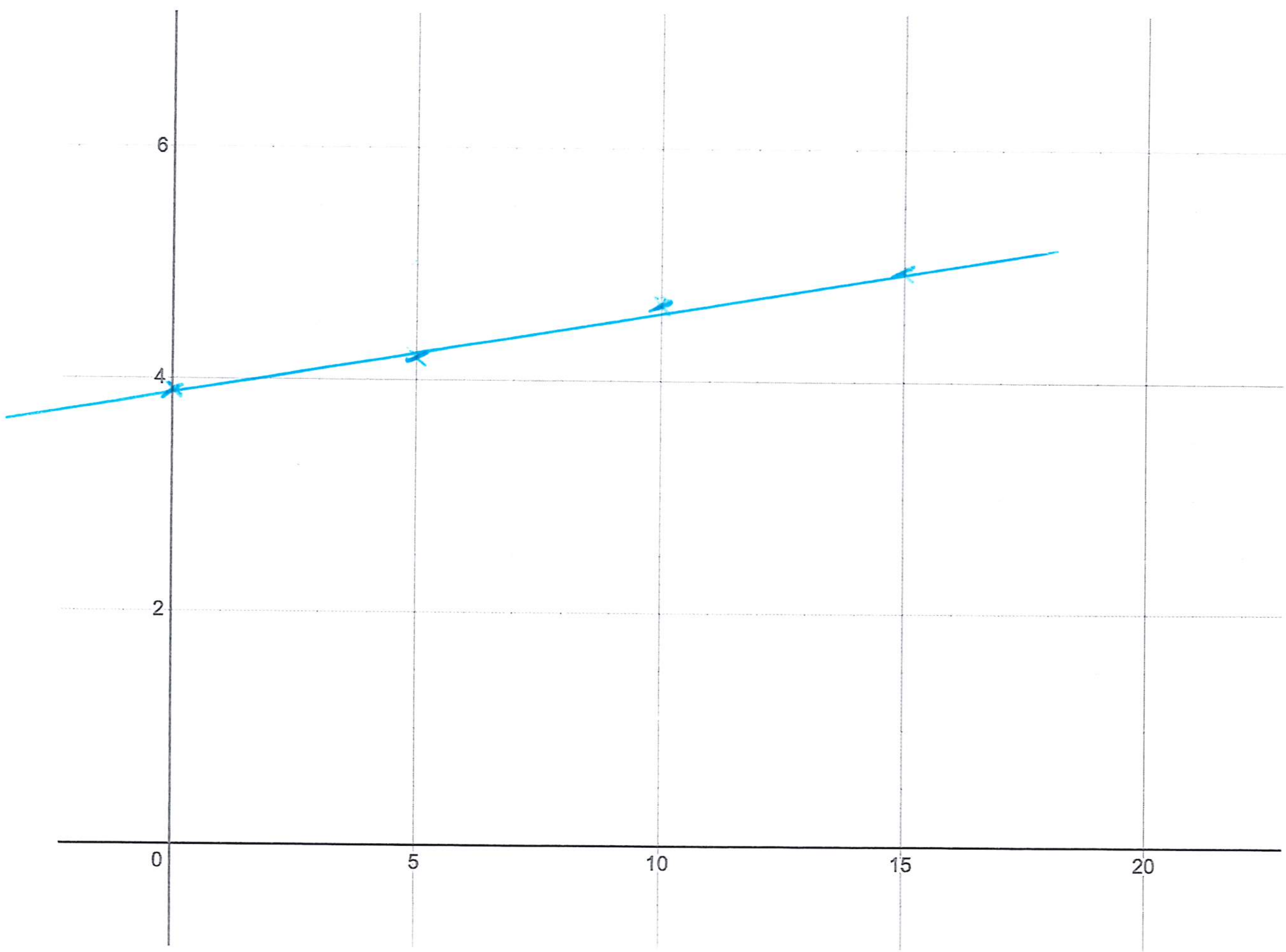
d) Find the expected value of the car in 2032 and comment on the validity of this value.

(2)

$$\log_{10}(V) = \log_{10}(a \times b^t)$$
$$= \log_{10}(a) + \log_{10}(b^t)$$
$$= \log_{10}(a) + t\log_{10}(b)$$
$$= \log_{10}(b)t + \log_{10}(a)$$

b)

t	0	5	10	15
$\log_{10}(V)$	3.99	4.29	4.59	4.90



9

Find the points of inflection of the curve with equation $y = \cot(2x)$

(7)

$$y = \cot(2x)$$

$$\frac{dy}{dx} = -2\operatorname{cosec}^2(2x)$$

$$\frac{d^2y}{dx^2} = -4\operatorname{cosec}(2x) \cdot -\operatorname{cosec}(2x)\cot(2x)2$$

$$= 8\operatorname{cosec}^2(2x)\cot(2x).$$

At an inflection point $\frac{d^2y}{dx^2} = 0$

Hence,

$$0 = 8\operatorname{cosec}^2(2x)\cot(2x)$$

$$\Rightarrow 0 = \cot(2x)\operatorname{cosec}^2(2x)$$

$$= \cot(2x)(1 + \cot^2(2x))$$

So

$\cot(2x) = 0$ or $1 + \cot^2(2x) = 0$

$\Rightarrow \frac{1}{\tan(2x)} = 0$ $\Rightarrow \cot^2(2x) = -1$ so no solutions

$$\Rightarrow x = \frac{\pi}{4}$$

Hence,

$y = \cot(2x)$ has inflection points at

$x = \frac{\pi}{4} + \frac{\pi n}{2} \quad , n \in \mathbb{N}$

10

a) Find, in ascending powers of x, up to the term including x^3, the binomial expansion if $(1+x)^{-\frac{1}{3}}$

(2)

b) A student approximates $\dfrac{1}{\sqrt[3]{7}}$ using the expansion found in (a) and the value $x = 6$. Explain what is wrong in doing this.

(1)

c) By using a suitable value of x and part (a), find an approximation to $\dfrac{1}{\sqrt[3]{7}}$.

(4)

a) $(1+x)^{-1/3} = 1 + \left(-\frac{1}{3}\right)(x) + \dfrac{\left(-\frac{1}{3}\right)\times\left(-\frac{4}{3}\right)x^2}{2} + \dfrac{\left(-\frac{1}{3}\right)\times\left(-\frac{4}{3}\right)\times\left(-\frac{7}{3}\right)x^3}{6}$

$\Rightarrow (1+x)^{-1/3} = 1 - \frac{1}{3}x + \frac{2}{9}x^2 - \frac{14}{81}x^3$ (+)

b) The expansion of $(1+x)^{-1/3}$ is only valid for $|x| < 1$ and $6 > 1$ so it is outside of the domain of validity.

c) Let $x = -\frac{1}{8}$, then, using LHS of (+)

$$(1+x)^{-1/3} = \left(1-\frac{1}{8}\right)^{-1/3} = \left(\frac{7}{8}\right)^{-1/3}$$

$$= \left(\frac{8}{7}\right)^{1/3}$$

$$= \frac{2}{\sqrt[3]{7}}$$

Substituting in $x = \frac{-1}{8}$ on the RHS of (+)

$$2\left(\frac{1}{\sqrt[3]{7}}\right) \approx 1 - \frac{1}{3}\left(-\frac{1}{8}\right) + \frac{2}{9}\left(-\frac{1}{8}\right)^2 - \frac{14}{81}\left(-\frac{1}{8}\right)^3$$

$$= 1.045476466$$

So

$$\frac{1}{\sqrt[3]{7}} \approx 0.522738233$$

11

The rate of change of the radius of a circle is inversely proportional to the radius cubed.

a) Show that the rate at which the area of the circle, A, changes satisfies $\dfrac{\mathrm{d}A}{\mathrm{dt}} = \dfrac{2\pi^2}{A}$

(4)

b) Explain why $\dfrac{\mathrm{d}A}{\mathrm{d}t} > 0$

(1)

$$\frac{dr}{dt} \propto \frac{1}{r^3}$$

but

$$\frac{dr}{dA} \times \frac{dA}{dt} = \frac{dr}{dt}$$

wlog let $k=1$, $\dfrac{dr}{dt} = \dfrac{1}{r^3}$

so since $A = \pi r^2 \Rightarrow \dfrac{dA}{dr} = 2\pi r$

$$\Rightarrow \frac{dr}{dA} = \frac{1}{2\pi r}$$

Hence $\dfrac{1}{2\pi r} \dfrac{dA}{dt} = \dfrac{1}{r^3}$

$$\Rightarrow \frac{dA}{dt} = \frac{2\pi r}{r^3}$$

$$= \frac{2\pi}{r^2}$$

$$= \frac{2\pi^2}{\pi r^2}$$

$$= \frac{2r^2}{A}$$

b) $r > 0 \Rightarrow \frac{1}{r^2} > 0$

$$\Rightarrow \frac{dr}{dt} > 0$$

$$\Rightarrow \frac{dA}{dt} > 0$$

12

Consider the circle $x^2 - 8x + y^2 - 8y + 7 = 0$

a) What is the centre, C, of the circle?

[1 mark]

b) Find the equation of the tangent to the circle at $P(8,7)$

[3 marks]

c) Another tangent to the circle at $Q(7,0)$ has equation $3x - 4y = 21$.

This tangent meets the tangent found in (a) at the point R.

Shape T is formed by removing the sector CPQ of the circle from the quadrilateral $CPRQ$.

Find the ratio

Perimeter of T : Area of T

[8 marks]

a) $x^2 - 8x + y^2 - 8y + 7 = 0$

$\Rightarrow (x-4)^2 + (y-4)^2 = 25$

So the centre is $(4,4)$

b) Gradient of $CP = \frac{3}{4}$ $\Rightarrow$ gradient of tangent is $-\frac{4}{3}$

Hence, equation of the tangent is of the form $y = -\frac{4}{3}x + C$, passing through $(8,7)$.

So, $7 = -\frac{4}{3} \times 8 + C$

$\Rightarrow c = 7 + \frac{4}{3} \times 8$

$= \frac{53}{3}$

Hence, equation of the tangent is

$y = -\frac{4}{3}x + \frac{53}{3}$

$\Rightarrow 3y + 4x = 53$

c) Solving $3x - 4y = 21$ and $3y + 4x = 53$ simultaneously gives the coordinates of R as $(11, 3)$.

Gradient CP $= \frac{3}{4}$

Gradient CQ $= -\frac{4}{3}$

Hence, CP and CQ are perpendicular, so given that CP meets the tangent at 90° and CQ meets the tangent at 90°, we must have a quadrilateral with four 90° angles.

$|PR| = \sqrt{(11-8)^2 + (3-7)^2}$

$= \sqrt{3^2 + 4^2}$

$= 5$

Hence, PR = 5 also, and so we have CPRQ is a square.

So perimeter of CPRQ is 20
and area is 25.

Arclength PQ $= 5 \times \frac{\pi}{2} = \frac{5\pi}{2}$

~~Arclength~~

Area of sector CPQ $= \frac{1}{2} \times 5^2 \times \frac{\pi}{2}$

$= \frac{25\pi}{4}$

∴ Perimeter of T $= 10 + \frac{5\pi}{2}$

$= \frac{20 + 5\pi}{2}$

Area of T $= 25 - \frac{25\pi}{4}$

$= \frac{25(4-\pi)}{4}$

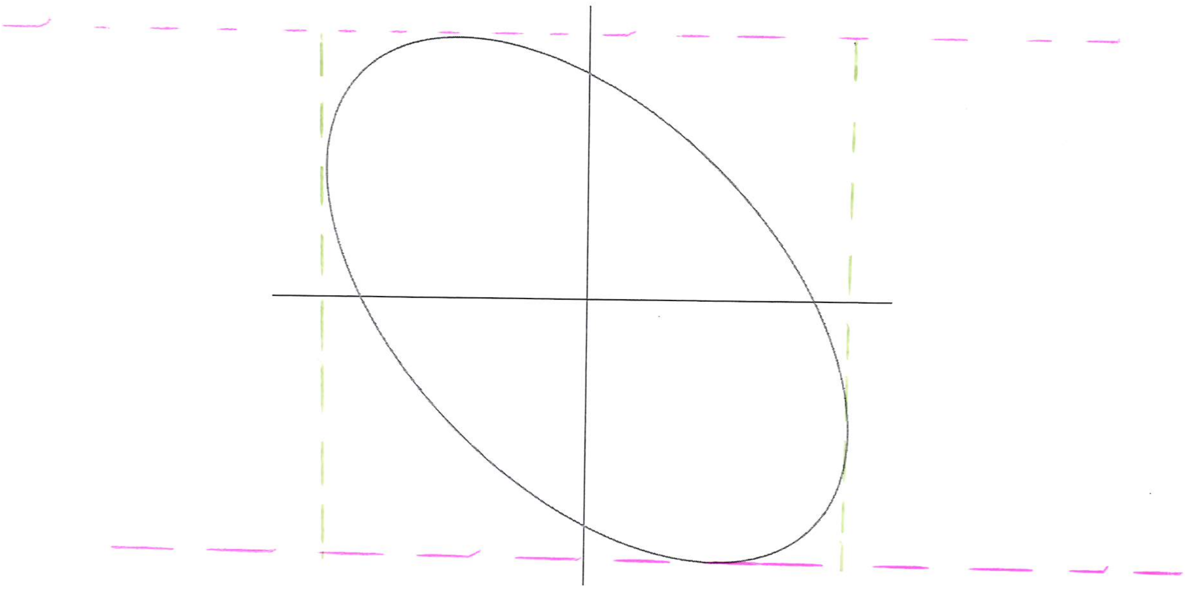
So the ratio

$P : T = \frac{20 + 5\pi}{2} : \frac{25}{4}(4-\pi)$

$= 2(4+\pi) : 5(4-\pi)$

13

The sketch below shows the curve with equation $x^2 + xy + y^2 = a$ for some constant a.



Find the area of the rectangular box that just contains the curve.

(10)

Horizontal edges of the box are bounded by the maximum and minimum values of the y-coordinates for points on the curve.

So, finding stationary points, differentiating with respect to x

$$\frac{d}{dx}\left[x^2+xy+y^2\right] = \frac{d}{dx}[a]$$

$$\Rightarrow \quad 2x + x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx}(x+2y) = -2x - y$$

Hence,

$$\frac{dy}{dx} = \frac{-2x-y}{x+2y}$$

At a stationary point, $\frac{dy}{dx} = 0$.

So, $-2x - y = 0 \Rightarrow y = -2x$

The point also satisfies the equation of the curve, hence,

$$x^2 - 2x \times x + (-2x)^2 = a$$

$$3x^2 = a$$

$$\Rightarrow x = \pm\sqrt{\frac{a}{3}}$$

When $x = \sqrt{\frac{a}{3}}$, $y = -2\sqrt{\frac{a}{3}}$

So the height of the box is $4\sqrt{\frac{a}{3}}$ (the curve is symmetric in height about the x-axis)

For the vertical edges, these are where the tangents to the curve are of the form $y = c$, $c \in \mathbb{R}$.

This occurs when $\frac{dy}{dx}$ is undefined. To this end, consider

$$x + 2y = 0$$

$$\Rightarrow x = -2y$$

then as it satisfies the equation of the curve too,

$$(-2y)^2 + (-2y)y + y^2 = a$$

$$3y^2 = a$$

$$y = \pm\sqrt{\frac{a}{3}}$$

When $y = \pm\sqrt{\frac{a}{3}}$, $x = \mp 2\sqrt{\frac{a}{3}}$

So, the width of the box is $4\sqrt{\frac{a}{3}}$

Hence, area of the box is $4\sqrt{\frac{a}{3}} \times 4\sqrt{\frac{a}{3}} = \frac{16a}{3}$