

a) Find the vector equation of the line through the points $A(1,2,4)$ and $B(3,5,1)$.
b) Show that the line found in (a) and the line with equation $\mathbf{r} = \begin{pmatrix} -2 \\ -2 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} -3 \\ -2 \\ -2 \end{pmatrix}$ are skew.

The roots of the equation $x^3 - 12x^2 + cx + d = 0$ form an arithmetic progression. Show that $4c + d = 128$.

a) Sketch on the same axes the functions with polar equation $r = \sin(\theta)$ and $r = \sqrt{3} \cos(\theta)$.
b) Find their point of intersection.

a) Show that $\frac{3}{r^2 + 3r} = \frac{1}{r} - \frac{1}{r + 3}$
b) Hence find an expression for the summation $\sum_{r=1}^n \frac{3}{r^2 + 3r}$.

a) Show that $\operatorname{arsinh}(x) = \ln(x + \sqrt{x^2 + 1})$
b) Solve the equation $3 \sinh^2(x) + \sinh(x) - 2 = \cosh^2(x)$

Let $z = 2 + 3i$ and $w = 3 - 4i$, Calculate and show on an Argand Diagram
i) zw
ii) $\frac{w}{z}$
iii) zw^*
Find also $|w|$ and $\arg(w)$

a) Find, without using calculus, the coordinates of the turning points of the function $f(x) = \frac{2x^2 - 5x - 3}{x^2 + 3x + 2}$
b) Hence, sketch $f(x)$ indicating the equations of any asymptotes.

a) Let $A = \begin{pmatrix} 1 & 3 \\ 4 & 13 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 3 \\ 3 & 5 \end{pmatrix}$. Verify for these matrices that $(AB)^{-1} = B^{-1}A^{-1}$. Prove also, that for general $n \times n$, non-singular matrices A and B that this result holds.

Prove, by induction that $11^n - 6^n$ is divisible by 5 for all natural numbers n .

Given that $z_1 = 2 - 3i$ is a root of the polynomial $p(z) = 2z^4 - 9z^3 + 24z^2 + 11z - 78$ express $p(z)$ in fully factorised form.

Using standard results find the Maclaurin series expansion of $y = e^{\sin(2x)}$ up to the third non-zero term.

A curve C is given by the equation $y = \sqrt{2x^2 - 3}$. What is the volume generated when the region enclosed by the curve, the x -axis and the lines $x = 1$ and $x = 3$ is rotated 2π radians around the x -axis.