AQA A-Level Further Mathematics Warmup - Paper 3MS 2025

A 1.5 kg mass is attached to a string OP of length 1 m where O is a fixed point. When P is vertically below O, at point Q, it is given a horizontal velocity of 5 ms-1.
When $\angle QOP = 60°$ find the velocity, v, of the particle and the tension, T, in the string.

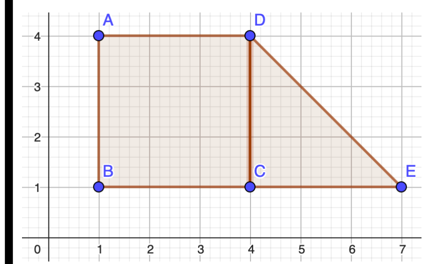
Prove that the exponential distribution $f(x) = \lambda e^{-\lambda x}$, with $x \geq 0$ has a mean of $\frac{1}{\lambda}$.

A discrete random distribution has probability distribution shown below. Given that $E(X) = 2.3$ find the values of a and b.

x	1	2	3	4
$P(X = x)$	0.4	a	b	0.2

What is the formula for calculating the χ^2 test statistics? What is the formula for Yate's correction, and when is this used?

Find the centre of mass of the lamina shown below.
If it is suspended from A, find the angle the vertical makes with the side AD



When would you use a $t-$test? And what is the formula for the test statistic?

A random variable Y has expected value 5 and variance 1.6.
Find the expected value and variance for the random variable $Y = 2x + 3$.

Derive the formula for the work done in stretching a string of natural length l to a length of $l + x$ given that the modulus of elasticity of the string is λ.

Derive the mean for a discrete uniform distribution $X \sim U(n)$. State the formula for the Var(X)

A car of mass 1200 kg is moving down a hill inclined at an angle θ where $\sin(\theta) = \frac{1}{30}$. The car is accelerating at 1.2ms^{-1} and the engine is working at a constant rate of 35 kW. Find the magnitude of the non-gravitational resistance to motion at the instant when the car is moving travelling at 5 ms^{-1}.

A uniform rod AB of mass 2000 grams is pivoted at A and held in equilibrium at an angle of $45°$ to the vertical by a force F applied at B, perpendicular to AB. Find the force F

Random events occur at a rate of 5 per minute.
a) Write the probability density function $f(t)$ and the cumulative density function $F(t)$ for the random variable T, the waiting time in minute between events.
b) What is the mean and variance of T.

A particle A of mass 2 kg is attached to the lower end of a light inextensible string with the upper end fixed at a point B. When the particle moves in a horizontal circular path, the string traces out the curved surface of a cone and makes an angle $60°$ with the downward vertical. The centre of the circular path lies 2 m directly below B

a) Find the tension in the string

b) Find the angular speed of the particle

A Poisson distribution is believed to have parameter $\lambda = 2.5$. The hypotheses $H_0 : \lambda = 2.5$ and $H_1 : \lambda > 2.5$ are tested at the $5\,\%$ level. A sample of size 7 is taken.
What is the critical value?
What is the probability of making a Type I error?

A Geiger counter detects radioactive decays at a mean rate of 25 per minute. Find the probability that in a given, randomly chosen minute, there are
i) 22 decays
ii) More than 27 decays

The time period t of a pendulum is modelled by the formula $t = km^{\alpha}l^{\beta}g^{\gamma}$ where k is a constant, m is the mass of the bob, l is the length and g is the acceleration dues to gravity.
Find the values of α, β and γ

A continuous random variable, X, has probability density function

$$f(x) = \begin{cases} -kx(x-4); & 0 \leq x \leq 4 \\ 0; & \text{otherwise} \end{cases}$$

for some positive constant k. Find k.

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By conservation of energy, total energy at P = total energy at Q. Hence,

$$\frac{3}{4}v^2 + \frac{3}{2}g(1 - \cos(60)) = \frac{1}{2} \times \frac{3}{2} \times 5^2$$
$$\frac{3}{4}v^2 = \frac{75}{4} - \frac{3}{4}g$$
$$v^2 = \frac{76}{5}$$

So $v = 3.90$ ms-1 to 2dp.

Apply $F = ma$ towards the centre, then

$$T = \frac{3}{2}g\cos(60) + \frac{\frac{3}{2} \times \frac{76}{5}}{1} = 30.15 \text{ N to 2dp.}$$

$$\mathrm{E}[X] = \int_{-\infty}^{\infty} x f(x)\,\mathrm{d}x = \int_0^{\infty} x\lambda \mathrm{e}^{-\lambda x} = \left[-x\mathrm{e}^{-\lambda x}\right]_0^{\infty} - \left[\frac{1}{\lambda}\mathrm{e}^{-\lambda x}\right]_0^{\infty} = \frac{1}{\lambda}$$

using integration by parts.

Probabilities must sum to one. Hence, we have

$$0.4 + a + b + 0.2 = 1$$
$$\Rightarrow \quad a + b = 0.4$$

Similarly, since $\mathrm{E}(X) = 2.3$ we have $2a + 3b = 1.1$. Solving these gives $a = 0.1$ and $b = 0.3$.

$$\chi^2 = \sum_i \frac{(O_i - E_i)}{E_i}$$

Yate's correction is used for 2×2 contingency tables and has the formula.

$$\chi^2_{\text{Yates}} = \sum \frac{(|O_i - E_i| - 0.5)^2}{E_i}$$

$$\frac{27}{2}\bar{x} = 9 \times \frac{5}{2} + \frac{9}{2} \times 5$$

so $\bar{x} = \dfrac{10}{3}$

$$\frac{27}{2}\bar{y} = 9 \times \frac{5}{2} + \frac{9}{2} \times 2$$

So $\bar{y} = \dfrac{7}{3}$.

The angle the side AD makes with the vertical is

$$\theta = \arctan\left(\frac{5}{7}\right) \approx 35.54°$$

Suppose a sample of size n is taken from a distribution. We use a $t-$test if the population variance is unknown and we only know the sample variance s^2. In this case the test statistic is $T = \dfrac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$ and it follows a $t-$distribution with $n - 1$ degrees of freedom.

$$\mathrm{E}(Y) = 2\mathrm{E}(X) + b = 2 \times 5 + 3 = 13$$

and

$$\mathrm{Var}(Y) = 2^2\mathrm{Var}(X) = 6.4$$

$$\text{Work} = \int_0^x T\,\mathrm{d}x = \int_0^x \frac{\lambda x}{l}\,\mathrm{d}x = \frac{\lambda}{l}\int_0^x x\,\mathrm{d}x = \frac{1}{2} \times \frac{\lambda}{l}\left[x^2\right]_0^x = \frac{\lambda}{2l}x^2$$

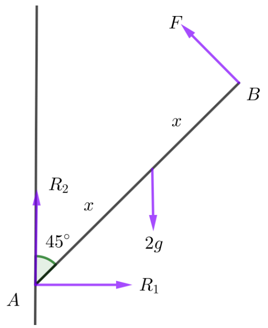
$$E[x] = \sum_{r=1}^{n} r \times \frac{1}{n} = \frac{1}{n}\frac{r(n+1)}{2} = \frac{n+1}{2}$$

$$\mathrm{Var}(x) = \frac{n^2 - 1}{12}$$

Let R be the non gravitational resistance to motion and T be the tractive force of the car. Using $P = Fv$, $T = 7000$ N. Applying $F = ma$ down the plane we have that

$$T - R + 1200g\sin(\theta) = 1200 \times 1.2$$

and so $R = 5952$.



Exponential distribution

$f(t) = 5\mathrm{e}^{-5t}$, $t \geq 0$

$F(t) = 1 - \mathrm{e}^{-5t}$, $t \geq 0$

Mean: $\dfrac{1}{\mu} = \dfrac{1}{5}$

Variance: $\dfrac{1}{\mu^2} = \dfrac{1}{25}$

a) Resolving vertically,

$$T\cos(6) - 2g = 0$$
$$\Rightarrow \quad T = \frac{2g}{0.5} = 39.2 \text{ N}$$

Under the null hypothesis, $Y \sim \mathrm{Po}(17.5)$. $P(Y \geq 25) = 0.0532$ and $P(Y \geq 26) = 0.0339$. Hence the critical value is 26. The probability of making a Type I error is 0.0339.

Let $X \sim \mathrm{Po}(25)$. Then

$$P(X = x) = \mathrm{e}^{-25}\frac{25^x}{x!}.$$

$$P(X = 22) = 0.0702$$

$$\mathrm{P}(X > 27) = 1 - \mathrm{P}(X \leq 27) = 1 - 0.7001 = 0.2998$$

Taking anticlockwise moments about A.

$$2gx\cos(45) - F \times 2x = 0$$
$$F = \frac{2g\cos 45}{2}$$

So, $= \dfrac{49\sqrt{2}}{10} \approx 6.92$ N

Taking dimensions

$[T] = [m]^{\alpha}[l]^{\beta}[g]^{\gamma}$ and so,

$T = M^{\alpha}L^{\beta}(LT^{-2})^{\gamma}$ taking k to be a dimensionless constant.

So we have,

$1 = -2\gamma$

$0 = \alpha$

$0 = \beta + \gamma$

So $\alpha = 0$, $\beta = \dfrac{1}{2}$ amd $\gamma = -\dfrac{1}{2}$. Hence,

$$t = k\sqrt{\frac{l}{g}}$$

b) Applying $F = ma$ towards the centre.

$T\sin(60) = 2a \Rightarrow a = \dfrac{T\sin(60)}{2}$

so $a = \dfrac{49\sqrt{3}}{5}$.

Now $\omega^2 = \sqrt{\dfrac{a}{r}}$, so

$$\omega = \sqrt{\frac{16.97}{1.5\tan 60}} \approx 2.55$$

$$\int_0^4 -kx(x-4)\,\mathrm{d}x = 1,$$

hence $\left[-\dfrac{k}{3}x^3 + 2kx^2\right]_0^4 = 2$

and so $k = \dfrac{3}{32}$