AQA AS-Level Further Mathematics Warmup - Paper 1 2026

Sketch $y = \dfrac{3x+2}{2x-5}$ stating the equations of the asymptotes and any intersections with the axes.	The polynomial $15x^2 - 23x + 4$ has roots α and β. Find a) $\alpha + \beta$ b) $\alpha\beta$ c) $\alpha^2 + \beta^2$ d) $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$	Find the vector equation of the line joining the points $A(3,2,1)$ to $B(5,-4,3)$.	Find the image of the point $P(2,5)$ after a reflection in the line $y = -x$ followed by an enlargement centre the origin, scale factor 2.	Sketch $y = \text{arsinh}(x)$ along with $y = \sinh(x)$ and find $y = \text{arsinh}(3)$
a) Sketch the locus represented by $\lvert z - 6 - 5\text{i}\rvert = 4$. **b)** Find also the maximum and minimum values of $\lvert z\rvert$.	Given that $z = 2 + i$ is a root of $p(z) = z^3 - z^2 - 7z + 15$ find the other roots.	Prove the hyperbolic identity $\cosh^2(x) - \sinh^2(x) = 1$	**a)** Show that $\dfrac{1}{r+1} - \dfrac{1}{r+2} = \dfrac{1}{r^2+3r+2}$ **b)** Hence, prove that $\displaystyle\sum_{r=1}^{n} \frac{1}{r^2+3r+2} = \frac{n}{2(n+2)}$	Sketch the conic with equation $9x^2 - 36x + 4y^2 - 8y + 39 = 0$
Find the volume generated when the graph of $y = \sqrt{2x+3}$ between $x = 1$ and $x = 3$ is rotated 2π radians around the $x-$axis.	Sketch $r = 3 + 2\sin(\theta)$	Find the invariant lines of the matrix $\mathbf{M} = \begin{pmatrix} -\frac{4}{5} & \frac{3}{5} \\ \frac{3}{5} & \frac{4}{5} \end{pmatrix}$	Find the acute angle between the vectors $\mathbf{u} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix}$	Find, without using the calculus the coordinates of the turning point of $f(x) = \dfrac{2}{(x-1)(x+2)}$
Prove by induction that $\displaystyle\sum_{r=1}^{n} r^2 + r = \frac{1}{3}n(n+1)(n+2)$	The polynomial $p(x) = 6x^3 - 7x^2 - 14x + 8$ has roots α, β and γ. Find the polynomial with roots $3\alpha + 1$, $3\beta + 1$ and $3\gamma + 1$.	What are the relationships between the cartesian coordinates (x, y) and the polar coordinates (r, θ). Find the Cartesian equation for $r^4 = \cos(\theta)\sin(\theta)$	Find the Maclaurin Series of $y = e^x \sin(x)$ up to the term in x^3.	When is the matrix $\mathbf{A} = \begin{pmatrix} 2a & a \\ 3 & 3a \end{pmatrix}$ singular?

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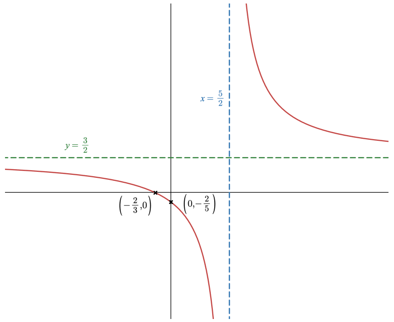
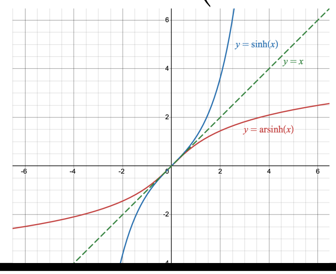


$$\alpha+\beta=\frac{23}{15}$$
$$\alpha\beta=\frac{4}{15}$$
$$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=\frac{409}{225}$$
$$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}=\frac{23}{4}$$

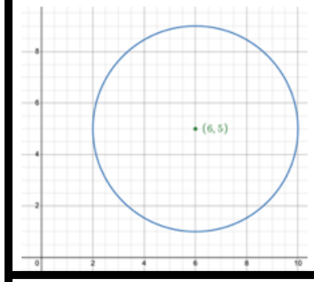
$$\mathbf{r}=\begin{pmatrix}3\\2\\1\end{pmatrix}+\lambda\begin{pmatrix}2\\-6\\2\end{pmatrix}$$

$$\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}2&0\\0&2\end{pmatrix}\begin{pmatrix}0&-1\\-1&0\end{pmatrix}\begin{pmatrix}2\\5\end{pmatrix}$$
hence the image, P' has coordinates $P'(-10,-4)$.

$$\operatorname{arsinh}(3)=\ln\left(3+\sqrt{10}\right)$$



$|z|_{\text{max}}=\sqrt{61}+4$
$|z|_{\text{min}}=\sqrt{61}-4$



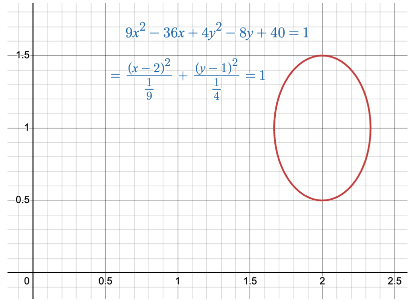
The roots are:

$$z_1=2+\mathrm{i}$$
$$z_2=2-\mathrm{i}$$
$$z_3=-3$$

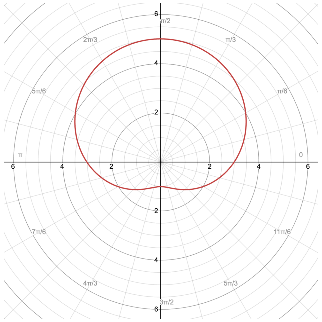
$$\begin{aligned}\cosh^2(x)-\sinh^2(x)&=\left(\frac{e^x+e^{-x}}{2}\right)^2-\left(\frac{e^x-e^{-x}}{2}\right)^2\\&=\frac{e^{2x}+2+e^{-2x}-\left(e^{2x}-2+e^{-2x}\right)}{4}\\&=\frac{4}{4}\\&=1\end{aligned}$$

a)
$$\frac{1}{r+1}-\frac{1}{r+2}=\frac{(r+2)-(r+1)}{(r+2)(r+1)}=\frac{1}{r^2+3r+2}$$

b) This is a method of differences question. As it says hence, you know you need to use the result established in (a).



$$\begin{aligned}V&=\pi\int_1^3\left(\sqrt{2x+3}\right)^2\,dx\\&=\pi\int_1^3 2x+3\,dx\\&=\pi\left[x^2+3x\right]_1^3\\&=14\pi\end{aligned}$$



Invariant lines satisfy
$$\begin{pmatrix}x'\\y'\end{pmatrix}=\begin{pmatrix}-\frac{4}{5}&\frac{3}{5}\\\frac{3}{5}&\frac{4}{5}\end{pmatrix}\begin{pmatrix}x\\mx+c\end{pmatrix}$$
with $y'=mx'+c$

Here $y=3x$ and $y=-\frac{1}{3}x+C$ are invariant lines.

Using $\mathbf{u}\cdot\mathbf{v}=|\mathbf{u}||\mathbf{v}|\cos(\theta)$ we have
$$\cos(\theta)=\frac{18}{\sqrt{26}\sqrt{14}}.$$
Hence, $\theta=19.36°$

We set $k=\frac{2}{(x-1)(x+2)}$ and rearrange to a quadratic in k. Set the discriminant of this quadratic to zero and solve for k. This value is the $y-$coordinate of the turning point.

Turning Point: $\left(-\frac{1}{2},-\frac{8}{9}\right)$

Proof.
Remember to state "Let $P(n)$ be the statement "$\sum_{r=1}^{n}r^2+r=\frac{1}{3}n(n+1)(n+2)$" at the beginning and end properly with a statement such as: "We have shown the result to be true for $n=1$ and if $P(k)$ is true then so is $P(k+1)$. Hence by the principle of mathematical induction the r $P(n)$ holds for all $n\in\mathbb{N}$.

Let $w=3x+1$ and rearrange to $x=\frac{w-1}{3}$.
Substitute into $p(x)$, expand and simplify to obtain a polynomial in w.
$p(w)=2w^3-13w^2-22w+105$

$$x=r\cos(\theta)$$
$$y=r\sin(\theta)$$
$$r=\sqrt{x^2+y^2}$$
$$\theta=\arctan\left(\frac{y}{x}\right)$$
Cartesian Form: $x^2+y^2=xy$

Using the standard results $e^x=1+x+\frac{x^2}{2}+\frac{x^3}{6}$ and $\sin(x)=x-\frac{x^3}{6}+\frac{x^5}{120}$ we have
$$e^x\sin(x)=x+x^2+\frac{x^3}{3}$$

$\mathbf{A}$ is singular when $\det(\mathbf{A})=0$. Hence,
$$\begin{aligned}0&=2a\times 3a-a\times 3\\&=6a^2-3a\\&=3a(2a-1)\end{aligned}$$
So, $\mathbf{A}$ is singular when $a=0$, and $a=\frac{1}{2}$.