

<p>A particle of mass 2 kg is acted on by a time dependent force of $F = 2t + 2 \sin(t)$. Find the impulse exerted by this force if the force is applied for 2π seconds.</p>	<p>Derive the mean for a discrete uniform distribution $X \sim U(n)$. State, and prove the formula for the $\text{Var}(X)$</p>	<p>The lunch choices of two year groups at school are tested for association. A total of 120 students in both year 12 and 13 are asked. In Year 12 38 chose burger, 62 pizza and 20 salad. In Year 13, 48 chose burger, 43 pizza and 29 salad. Perform a χ^2 test at 5% for association.</p>	<p>A CRV X, has probability density function given by $f(x) = \begin{cases} kx^2; & 0 \leq x \leq 4 \\ 0; & \text{otherwise} \end{cases}$ Find the constant k, $P(X \geq 3)$ and $E(X)$ and $\text{Var}(X)$</p>	<p>A body moving on a horizontal circular path of radius r with a constant angular velocity has: speed - acceleration - centripetal force -</p>										
<p>A particle P of mass 2 kg is moving at a speed of 4 ms^{-1} collides with a particle Q of mass 3 kg which is moving at a speed of 2 ms^{-1} in the same direction as P. Given that the coefficient of restitution between P and Q is $\frac{1}{3}$ find the speeds of P and Q after the collision.</p>	<p>Define the Type I and Type II errors that can be made during a hypothesis test?</p>	<p>A Poisson distribution is believed to have a parameter of 8. A sample is taken to determine whether the parameter is different to this. State the null and alternative hypotheses for this test. what is the probability of making a Type I error at the 5% significance level?</p>	<p>An object of mass 2 kg is attached to the end of a light elastic string of length 1 m. The other end of the string is fixed to a point A and released. The modulus of elasticity of the string is 60 N. Find the extension when it reaches maximum speed, and hence the distance below A at this point.</p>	<p>A Geiger counter detects radioactive decays at a mean rate of 35 per minute. Find the probability that in a given, randomly chosen minute, there are i) 17 decays ii) More than 26 decays</p>										
<p>A CRV X, has probability density function given by $f(x) = \begin{cases} \frac{1}{4}x^3; & 0 \leq x \leq a \\ 0; & \text{otherwise} \end{cases}$ Find the constant a and the median value M, of X.</p>	<p>Find the work done when a light elastic string of natural length 1.0 m and modulus of elasticity 100 N is stretched from a length of 1.1 m to 1.3m.</p>	<p>A sample of size $n = 25$ is taken and the sample mean is found to be 23.2. Given that the population variance is $\sigma = 3.4$ find the 88% confidence interval.</p>	<p>Let X be a discrete random variable with $E(X) = 3.25$ and $\text{Var}(X) = \frac{1}{2}$. Find $E(2X + 3)$ and $\text{Var}(2X + 3)$. Find also $\text{Var}(2X)$ and $\text{Var}(X + X)$</p>	<p>A particle P of mass 2kg is moving at a speed of 5 ms^{-1} collides with a particle Q of mass 3kg which is moving at a speed of 2 ms^{-1} in the opposite direction to P. Given that after the collision P moves with speed 1 ms^{-1}, find the speed of Q after the collision.</p>										
<p>Chris pulls a crate with a rope parallel to the horizontal floor with a constant tension of 30 N. The crate passes points A and B which are 6 m apart in a time of 2 seconds with speeds 3 ms^{-1} and 5 ms^{-1} respectively. Find the average power.</p>	<p>A discrete random distribution has probability distribution shown below. Given that $E(X) = 2.3$ find the values of a and b.</p> <table border="1" data-bbox="497 1377 884 1445"> <tbody> <tr> <td>x</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> </tr> <tr> <td>$P(X = x)$</td> <td>0.4</td> <td>a</td> <td>b</td> <td>0.2</td> </tr> </tbody> </table>	x	1	2	3	4	$P(X = x)$	0.4	a	b	0.2	<p>A ball of mass 200 g is attached to a light inextensible string of length 125 cm. One end of the string is fixed to a smooth horizontal table and moves in a circular path with linear speed 6 ms^{-1}. Find the acceleration of the ball and the tension in the string.</p>	<p>Derive the formula for the work done in stretching a string of natural length l to a length of $l + x$ given that the modulus of elasticity of the string is λ.</p>	<p>The excess pressure, P, inside a soap bubble is given by the formula $P = 4r^\alpha s^\beta$ where r is the radius, s is the surface tension and α and β are constants. Given that surface tension is defined to be force per unit length determine the values of α and β in the model for P.</p>
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$I = \int_0^{2\pi} F dt$ $= \int_0^{2\pi} 2t + 2 \sin(t) dt$ $= [t^2 + 2 \cos(t)]_0^{2\pi}$ $= (4\pi^2 + 2) - (0 - 2)$ $= 4\pi^2$	$E[x] = \sum_{r=1}^n r \times \frac{1}{n}$ $= \frac{1}{n} r(n+1)$ $= \frac{n+1}{2}$ $\text{Var}(x) = \frac{n^2 - 1}{12}$	<p>H_0 : no association H_1 : there is an association</p> <p>χ^2 test statistic = 6.25394716 2 degrees of freedom. Critical value is 5.991 Hence, there is sufficient evidence to reject H_0 and conclude that there is a possible association between year group</p>	$\int_0^4 kx^2 dx = 1 \Rightarrow k = \frac{3}{64}$ $P(X \geq 2) = \int_2^4 \frac{3}{64} x^2 dx = \frac{7}{8}$ $E(X) = \int_0^4 \int_2^4 \frac{3}{64} x^3 dx = 3$ $E(X^2) = \int_2^4 \frac{3}{64} x^4 dx = \frac{48}{5}$ $\text{Var}(X) = E(X^2) - (E(X))^2 = 7$	<p>Speed: $v = r\omega$, constant along the tangent.</p> <p>Acceleration: $a = r\omega^2 = \frac{v^2}{r}$ towards the centre.</p> <p>Centripetal Force: $F = mr\omega^2 = m \frac{v^2}{r}$</p>
<p>Let the speed after of P be u and the speed after of Q be v. Using conservation of momentum $2 \times 4 + 3 \times 6 = 2u + 3v$ and so $2u + 3v = 14$. Using Newton's law of restitution $e = \frac{\text{speed of separation}}{\text{speed of approach}}$ and so $\frac{1}{3} = \frac{v-u}{4-2} \Rightarrow \frac{2}{3} = v-u$ Solving these we obtain $u = \frac{12}{5} \text{ ms}^{-1}$ and $v = \frac{46}{15} \text{ ms}^{-1}$</p>	<p>A Type I error is when a null hypothesis which is true is rejected (sometimes called a false positive). A type II error is when a null hypothesis which is false is not rejected (sometimes called a false negative)</p>	<p>$H_0 : \lambda = 16$ $H_1 : \lambda \neq 16$</p> <p>$P(\text{Type I error}) = 0.0137 + 0.0172 = 0.0309$</p>	<p>Applying $F = ma$ and using the fact that at maximum speed $a = 0$ we find that the tension T in the string satisfies $T = 2g$. Using Hooke's law $\frac{60x}{l} = 2g$ and so $x = \frac{2g}{60} = 0.327$. So the particle is 1.327 m below A.</p>	<p>Let $X \sim \text{Po}(35)$. Then</p> $P(X = x) = e^{-20} \frac{20^x}{x!}$ $P(X = 17) = 0.000315$ $P(X > 26) = 1 - P(X \leq 26) = 0.9514$
$\int_0^a \frac{1}{4} x^3 dx = 1 \Rightarrow a = 2$ $\int_0^M \frac{1}{4} x^3 dx = \frac{1}{2}$ $\Rightarrow \frac{1}{16} M^4 = \frac{1}{2}$ $\Rightarrow M \approx 1.682$	<p>Work done $= \frac{\lambda}{2l} (x_2^2 - x_1^2)$ So work done = $= \frac{100}{2} (0.3^2 - 0.1^2) = 4 J$</p>	<p>For an 88% confidence interval we need 6% in each tail. This gives a z value of 1.5547. Hence the limits of the confidence interval are $\bar{x} \pm 1.5547 \times \frac{\sigma}{\sqrt{n}}$ and so the interval is (22.143, 24.257).</p>	$E(2X + 3) = 2E(X) + 3 = 9.5$ $\text{Var}(2X + 3) = 2^2 \text{Var}(X) = 4 \times \frac{1}{2} = 2$ $\text{Var}(2X) = 2^2 \text{Var}(X) = 4 \times \frac{1}{2} = 2$ $\text{Var}(X + X) = \frac{1}{2} + \frac{1}{2} = 1$	<p>Let the speed of Q after the collision be v. Then, $2 \times 5 + 3 \times (-2) = 2 \times 1 + 3 \times v$ by conservation of momentum. So $v = \frac{2}{3} \text{ ms}^{-1}$</p>
<p>Average power = work done divided by the time taken. Work done = force \times distance. So average power = $\frac{30 \times 6}{2} = \frac{180}{2} = 90 \text{ W}$</p>	<p>Probabilities must sum to one. Hence, we have $0.4 + a + b + 0.2 = 1 \Rightarrow a + b = 0.4$ Similarly, since $E(X) = 2.3$ we have $2a + 3b = 1.1$. Solving these gives $a = 0.1$ and $b = 0.3$.</p>	$a = \frac{v^2}{r}$ $= \frac{6^2}{1.25}$ $= 28.8 \text{ ms}^{-2}$ <p>So, applying $F = ma$ we have $T = ma = 0.2 \times 28.2 = 5.76 \text{ N}$</p>	$\text{Work} = \int_0^x T dx$ $= \int_0^x \frac{\lambda x}{l} dx$ $= \frac{\lambda}{l} \int_0^x x dx$ $= \frac{1}{2} \times \frac{\lambda}{l} [x^2]_0^x$ $= \frac{\lambda}{2l} x^2$	<p>Using dimensional analysis, $ML^{-1}T^{-2} = L^\alpha (MT^{-2})^\beta$ Comparing powers, $1 = \beta$ $-1 = \alpha$ $-2 = -2\beta$ Hence, $P = 4r^{-1}s$</p>