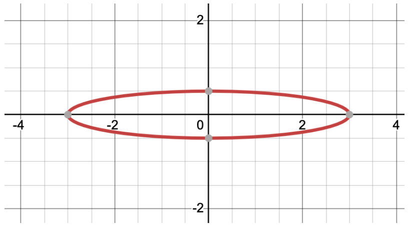
AQA A-Level Further Mathematics Warmup - Paper 2 2026

Sketch the conic section $\frac{x^2}{9} + 4y^2 = 1$	Find the vector equation of the line passing through $A(4,4,-2)$ and $B(5,2,3)$. What is the Cartesian equation in this case?	For what values of a is the matrix $\mathbf{N} = \begin{pmatrix} a-3 & 5 \\ 1 & a+1 \end{pmatrix}$ singular?	Find the invariant lines of the matrix $\mathbf{M} = \begin{pmatrix} -\frac{4}{5} & \frac{3}{5} \\ \frac{3}{5} & \frac{4}{5} \end{pmatrix}$	For $I_n = \int x^n e^{-x}\, dx$ the show that the reduction formula $I_n = nI_{n-1} - x^n e^{-x}$ holds. Use this to find $I_3 = \int x^3 e^{-x} dx$.
Define the scalar product for two vectors. Find the acute angle between the vectors $\mathbf{u} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix}$	Setup a matrix method to solve the simultaneous equations $3x + y + z = 11$ $2x + y + 3z = 17$ $x + y + z = 7$	Sketch $r = 3 + 2\cos(\theta)$	Find using the method of differences $\sum_{r=1}^{n} \frac{1}{r(r+2)}$	Solve $z^3 = 1$
Sketch $y = \lvert (x-3)(x-2)(x+2) \rvert$	Find the mean value of the function $y = \frac{1}{\sqrt{9+x^2}}$ between the lines $x = \ln(9)$ and $x = \ln(16)$	Find $I = \int \frac{7x^2 + 14}{(2x-1)(x^2+5)}\, dx$	Find, without using the calculus the coordinates of the turning point of $f(x) = \frac{2}{(x-1)(x+2)}$	Find the Maclaurin series expansion of $2e^x + e^{-x}$ up to the term involving x^3
For the complex number $z = \cos(\theta) + i\sin(\theta)$ state the relations relating powers of z and θ	Find the intersection point of the line $\mathbf{r} = \begin{pmatrix} 4 \\ 9 \\ 14 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 8 \\ 16 \end{pmatrix}$ and the plane $2x + 3y + 2z = 3$	Find using the method of differences $\sum_{r=1}^{n} \frac{1}{r(r+2)}$	Sketch $f(x) = \frac{x+1}{6x^2 + 10x - 4}$	Find the eigenvalues and eigenvectors of the matrix $\begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}$

AQA A-Level Further Mathematics Warmup - Paper 2 2026



$$\overrightarrow{AB} = \begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix}$$

$$\mathbf{r} = \begin{pmatrix} 4 \\ 4 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix}$$

$$x - 4 = \frac{4 - y}{2} = \frac{z + 2}{5}$$

If a matrix is singular then it's determinant is zero.

$$|\mathbf{N}| = 0$$

$$\Rightarrow \quad (a - 3)(a + 1) - 5 = 0$$

$$a^2 - 2a - 8 = 0$$

So $\mathbf{N}$ is singular for $a = 2$ or $a = 4$

Invariant lines satisfy

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -\frac{4}{5} & \frac{3}{5} \\ \frac{3}{5} & \frac{4}{5} \end{pmatrix} \begin{pmatrix} x \\ mx + c \end{pmatrix}$$

with $y' = mx' + c$

Here $y = 3x$ and $y = -\frac{1}{3}x + C$ are invariant lines.

$$-e^{-x}(x^3 + 3x^2 + 6x + 6) + C$$

$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos(\theta)$ where

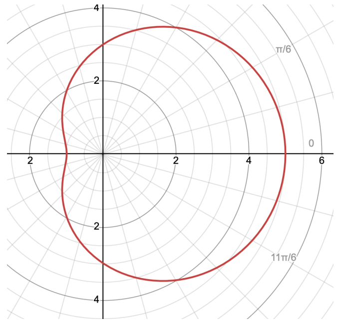
$\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$

Using $\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}||\mathbf{v}|\cos(\theta)$ we have $\cos(\theta) = \frac{18}{\sqrt{26}\sqrt{14}}$.

Hence, $\theta = 19.36^\circ$

$$\begin{pmatrix} 3 & 1 & 1 \\ 2 & 1 & 3 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 11 \\ 17 \\ 7 \end{pmatrix}$$

$x = 2,\ y = 1$ and $z = 4$



$$\frac{1}{r(r+2)} = \frac{1}{2r} - \frac{1}{2(r+2)}$$

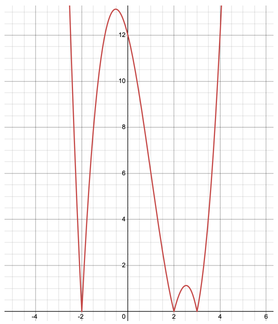
then using the method of differences

$$\sum_{r=1}^{n} \frac{1}{r(r+2)} = \frac{3}{4} - \frac{2n+3}{2(n+1)(n+2)}$$

$$z = 1$$

$$z = \frac{-1 + i\sqrt{3}}{2}$$

$$z = \frac{-1 - i\sqrt{3}}{2}$$



$$M = \frac{1}{\ln(16) - \ln(9)} \int_{\ln(9)}^{\ln(16)} \frac{1}{\sqrt{9 + x^2}} \, dx$$

$$= \frac{1}{\ln(16) - \ln(9)} \left[\operatorname{arsinh}\left(\frac{x}{4}\right) \right]_{\ln(9)}^{\ln(16)}$$

$$= \frac{1}{\ln(16) - \ln(9)} \left(\operatorname{arsinh}\left(\frac{\ln(4)}{3}\right) - \operatorname{arsinh}\left(\frac{\ln(3)}{3}\right) \right)$$

$$\approx 0.2568$$

$$\frac{7x^2 + 14}{(2x - 1)(x^2 + 5)} = \frac{3}{2x + 1} + \frac{2x - 1}{x^2 + 5}$$

So,

$$\int \frac{7x^2 + 14}{(2x - 1)(x^2 + 5)} \, dx = \int \frac{3}{2x - 1} + \frac{2x - 1}{x^2 + 5} \, dx$$

$$= \int \frac{3}{2x - 1} + \frac{2x}{x^2 + 5} - \frac{1}{x^2 + 5} \, dx$$

$$= \frac{3}{2} \ln|2x - 1| + \ln|x^2 + 5|$$

$$- \frac{1}{\sqrt{5}} \arctan\left(\frac{x}{\sqrt{5}}\right)$$

We set $k = \frac{2}{(x - 1)(x + 2)}$ and rearrange to a quadratic in k. Set the discriminant of this quadratic to zero and solve for k. This value is the $y-$coordinate of the turning point.

Turning Point: $\left(-\frac{1}{2}, -\frac{8}{9}\right)$

Using standard results

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots$$

$$e^{-x} = 1 - x + \frac{x^2}{2} - \frac{x^3}{6} + \cdots$$

So,

$$2e^x + e^{-x} = 2\left(1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots\right)$$

$$+ \left(1 - x + \frac{x^2}{2} - \frac{x^3}{6} + \cdots\right)$$

$$= 3 + x + \frac{3}{2}x^2 + \frac{1}{6}x^3$$

$$z^n + \frac{1}{z^n} = 2\cos(n\theta)$$

$$z^n - \frac{1}{z^n} = 2i\sin(n\theta)$$

Substitute a general point on the line into the equation of the plane as the intersection point satisfies both. Then,

$$2(4 + 2\lambda) + 3(9 + 8\lambda) + 2(14 + 16\lambda) = 3$$

$$60\lambda = -6$$

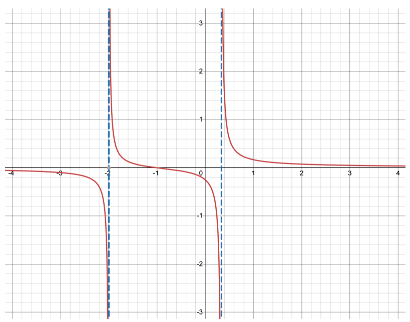
$$\lambda = -1$$

When, $\lambda = 1$ we have the point

$$\frac{1}{r(r+2)} = \frac{1}{2r} - \frac{1}{2(r+2)}$$

then using the method of differences

$$\sum_{r=1}^{n} \frac{1}{r(r+2)} = \frac{3}{4} - \frac{2n+3}{2(n+1)(n+2)}$$



$$\lambda_1 = 2,\ \mathbf{v}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\lambda_2 = 1,\ \mathbf{v}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$