Dr Bennison Maths Dr Bennison Maths

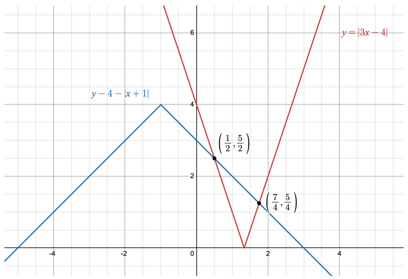
AQA A-Level Mathematics Warmup - Paper 1 2026

<table>
<tr>
<td>Differentiate $y = \sin(x)$ from first principles.</td>
<td>Sketch on the same axes $y = |3x - 4|$ and $y = 4 - |x - 1|$ and find their points of intersection.</td>
<td>Let $p(x) = 2x^4 + 5x^3 + ax^2 + bx + 45$
Given that $(x + 5)$ and $(x - 3)$ are factors, find a and b and hence fully factorise $p(x)$.</td>
<td>Find, in exact form, $\int_4^6 \frac{2x+8}{x^2+4x+3}\,dx$</td>
<td>Rationalise the denominator for $\frac{2+\sqrt{3}}{3+2\sqrt{3}}$</td>
</tr>
<tr>
<td>Find the equation of the tangent to $y = x\cos(x)$ at the point where $x = \frac{\pi}{2}$</td>
<td>The points $A(3,4)$ and $B(12,16)$ are the end points of the diameter of a circle.
Find the equation of the circle.</td>
<td rowspan="2">a) Find and evaluate $\frac{dy}{dx}$ at the points where $x = 1$ for the curve defined implicitly by $3x^2 + 2xy - 6y^2 + 5x = 4$

b) Find the equations of the tangents at these points and then find the intersection point of these two tangents.</td>
<td>Sketch $y = \sin(x)$ and $y = \text{cosec}(x)$ for $-2\pi \le x \le 2\pi$ on the same axes.</td>
<td>Where does the line perpendicular to $y = 2x + 5$ and passing through the point $(-4,4)$ cross the $x-$axes.</td>
</tr>
<tr>
<td>Find, up to the term in x^3 the binomial expansion of $\frac{1}{\sqrt{9+2x+}}$.</td>
<td>Express $y = 7\cos(x) + 4\sin(x)$ in the form $R\cos(x - \alpha)$</td>
<td>The 6th term of a geometric sequence is 1 and the 10th terms is $\frac{1}{625}$.
Find S_∞</td>
<td>Show that the derivative of $y = \tan(x)$ is $\frac{dy}{dx} = \sec^2(x)$. Hence, find $\frac{d}{dx}\left[\tan(2x^2 + 3x)\right]$</td>
</tr>
<tr>
<td>Find $\frac{dy}{dx}$ for $y = \frac{\cos(x)}{\sqrt{2x+1}}$</td>
<td>Find $\int 2x(3x^2+5)^{\frac{3}{2}}\,dx$</td>
<td>Prove that the product of two consecutive odd numbers is one less than a multiple of 4.</td>
<td>Find the values of k for the quadratic $(2+k)x^2 + (1-k)x + 2 = 0$ to have a repeated root.</td>
<td>Using the compound angle formulae derive the double angle formulae.</td>
</tr>
</table>

AQA A-Level Mathematics Warmup - Paper 1 2026

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$= lim_{h\to 0}\frac{sin(x+h)-sin(x)}{h}$$
$$= \lim_{h\to 0}\frac{\sin(x)\cos(h)+cos(x)\sin(h)-sin(x)}{h}$$
$$= \lim_{h\to 0}\frac{\sin(x)\left(1-\frac{h^2}{2}\right)+cos(x)h-sin(x)}{h}$$
$$= \lim_{h\to 0}-\frac{h}{2}\sin(x)+\cos(x)$$
$$= \cos(x)$$



By the factor theorem, $p(-5)=0$ and so $25a-5b=-670$ and also $p(3)=0$ so, $9a+3b=-342$. Solving these we obtain $a=-31$ and $b=-21$.
Hence,
$$p(x) = 2x^4+5x^3-31x^2-21x+45$$
$$= (x+5)(x-3)(2x^2+x-3)$$
$$= (x+5)(x-3)(2x+3)(x-1)$$

Using partial fractions $\dfrac{2x+8}{x^2+4x+3} = \dfrac{3}{x+1}-\dfrac{1}{x+3}$.
Hence,
$$\int_4^6 \frac{2x+8}{x^2+4x+3}\,dx = \int_4^6 \frac{3}{x+1}-\frac{1}{x+3}\,dx$$
$$= \left[3\ln|x+1|-\ln|x+3|\right]_4^6$$
$$= 3\ln|7|-\ln|9|-3\ln|5|+\ln|7|$$
$$= \ln|2401|-\ln|1125|$$
$$= \ln\left|\frac{2401}{1125}\right|$$

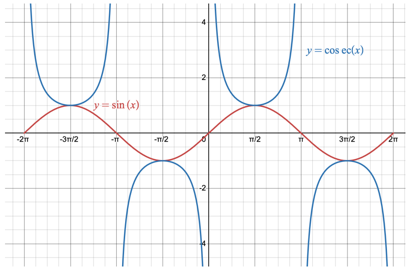
$$\frac{2+\sqrt{3}}{3+2\sqrt{3}} = \frac{2+\sqrt{3}}{3+2\sqrt{3}}\times\frac{3-2\sqrt{3}}{3-2\sqrt{3}}$$
$$= \frac{2\times 3-2\times 2\sqrt{3}+\sqrt{3}\times 3-\sqrt{3}\times 2\sqrt{3}}{3\times 3-3\times 2\sqrt{3}+2\sqrt{3}\times 3-2\sqrt{3}\times 2\sqrt{3}}$$
$$= \frac{6-4\sqrt{3}+3\sqrt{3}-6}{9-6\sqrt{3}+6\sqrt{3}-12}$$
$$= \frac{-\sqrt{3}}{-3}$$
$$= \frac{\sqrt{3}}{3}$$

$$\frac{dy}{dx} = \cos(x)-x\sin(x)$$
When $x=\dfrac{\pi}{2}$, $\dfrac{dy}{dx}=-\dfrac{\pi}{2}$. Since $y=0$ at $x=\dfrac{\pi}{2}$ the equation of the tangent is $y=-\dfrac{\pi}{2}x+\dfrac{\pi^2}{4}$

Midpoint:
$$\left(\frac{3+12}{2},\frac{4+16}{2}\right) = \left(\frac{15}{2},10\right)$$
Radius:
$$\sqrt{\left(12-\frac{15}{2}\right)^2+(16-10)^2} = \frac{15}{2}$$
Hence, circle is
$$\left(x-\frac{15}{2}\right)^2+(y-10)^2 = \frac{225}{4}$$

Differentiating implicitly:
$$6x+2x\frac{dx}{dy}+2y-12y\frac{dx}{dy}+5=0$$
and so rearranging
$$\frac{dy}{dx} = -\frac{5+2y+6x}{2x-12y}.$$
When $x=1$,
$$3+2y-6y^2+5=4$$
$$\Rightarrow\quad 6y^2-2y-4=0$$
$$\Rightarrow\quad (3y+2)(2y-2)=0$$
So points are $(1,1)$ and $\left(1,-\dfrac{2}{3}\right)$.

Tangents at these points are
$$y=-\frac{13}{10}x-\frac{3}{10}$$
$$y=-\frac{29}{30}x+\frac{3}{10}.$$
Hence, the point of intersection has $x-$coordinate $\dfrac{9}{34}$.



The gradient of the perpendicular line is $-\dfrac{1}{2}$ and so the perpendicular line is $y=-\dfrac{1}{2}x+2$ which crosses the $x-$axes at B

$$\frac{1}{\sqrt{9+2x}} = (9+2x)^{-\frac{1}{2}}$$
$$= \left[9\left(1+\frac{2x}{9}\right)\right]^{-\frac{1}{2}}$$
$$= \frac{1}{3}\left(1+\frac{2x}{9}\right)^{-\frac{1}{2}}$$
$$= \frac{1}{3}-\frac{x}{27}+\frac{x^2}{162}-\frac{5x^3}{4755}$$

Using the identity for $\cos(A-B)$ with $A=x$ and $B=\alpha$ we find that $4=R\sin(\alpha)$ and $7=R\cos(\alpha)$. Hence $R=\sqrt{7^2+4^2}=\sqrt{65}$ and $\alpha=\arctan\left(\dfrac{4}{7}\right)=0.519$. So
$$y=\sqrt{65}\cos\left(x-0.519\right)$$

Using the nth term formula for a geometric sequence:
$$ar^5=1$$
$$ar^9=\frac{1}{625}.$$
Solving these we obtain $r=\dfrac{1}{5}$ and $a=3125$.
Hence, $S_\infty = \dfrac{a}{1-r} = \dfrac{3125}{1-\frac{1}{5}} = \dfrac{15625}{4}$

If $y=\tan(x)=\dfrac{\sin(x)}{\cos(x)}$ we can use the quotient rule to find
$$\frac{dy}{dx} = \frac{\cos(x)\cos(x)-\sin(x)(-\sin(x))}{\cos^2(x)}$$
$$= \frac{\cos^2(x)+\sin^2(x)}{\cos^2(x)}$$
$$= \sec^2(x)$$
Hence,
$$\frac{d}{dx}\left[\tan(2x^2+3x)\right] = (4x+3)\sec^2(2x^2+3x)$$

Let $u=\cos(x)$ so $\dfrac{du}{dx}=-\sin(x)$ and also $v=(2x+1)^{\frac{1}{2}}$ so
$$\frac{dv}{dx} = (2x+1)^{-\frac{1}{2}}\times\frac{1}{2}\times 2$$
$$= \left(2x+1\right)^{-\frac{1}{2}}$$
Using the quotient rule
$$\frac{dy}{dx} = \frac{-2x\sin(x)-\sin(x)-\cos(x)}{(2x+1)^{\frac{3}{2}}}$$

Let $u=3x^2+5$ then $\dfrac{du}{dx}=6x$ and so $dx=\dfrac{du}{6x}$. So, integrating by substitution:
$$\int 2xu^{\frac{3}{2}}\frac{du}{6x} = \frac{1}{3}\int u^{\frac{3}{2}}\,du$$
$$= \frac{1}{3}u^{\frac{5}{2}}\times\frac{2}{5}+C_1$$
$$= \frac{2}{15}\left(3x^2+5\right)^{\frac{5}{2}}+C_2$$

Let the two odd numbers be $2n+1$ and $2n+3$ as they are consecutive.

Then,
$$(2n+1)(2n+3) = 4n^2+8n+3$$
$$= 4(n^2+2n)-1$$
which is one less than a multiple of 4.

The discriminant of $(2+k)x^2+(1-k)x+2=0$ is $k^2-10k-15$. For a repeated real root the discriminant is equal to zero. This occurs at $k=5-2\sqrt{10}$ and $5+2\sqrt{10}$

$$\sin(2x) = 2\sin(x)\cos(x)$$
$$\cos(2x) = \cos^2(x)-\sin^2(x)$$
$$= 2\cos^2(x)-1$$
$$= 1-2\sin^2(x)$$
$$\tan(2x) = \frac{2\tan(x)}{1-\tan^2(x)}$$