


<p>Find the cartesian equation for the curve with parametric equations</p> $x = \sin(\theta),$ $y = 3 \sin(2\theta).$	<p>A ball is projected upwards at a speed of <math>5 \text{ ms}^{-1}</math> at an angle of <math>25^\circ</math>. Find the vertical and horizontal components of their initial velocity.</p>	<p>Find the magnitude of the vector <math>\mathbf{v} = 3\mathbf{i} + 5\mathbf{j}</math>. Find also the angle it makes with the vector <math>\mathbf{i}</math>. What is the unit vector parallel to <math>\mathbf{v}</math>?</p>	<p>Let <math>f(x) = 3x + 6</math> and <math>g(x) = \frac{3x - 1}{2x^2 - 1}</math>. Solve <math>gf(x) = 0</math></p>	<p>Given that the coefficient of <math>x^3</math> in the expansion of <math>(2 + ax)^6 = 20</math> is 20. Find <math>a</math>.</p>
<p>An arithmetic sequence has tenth term 35 and fifteenth term 50. Find the sum of the first 50 terms.</p>	<p><math>t</math> is given that <math>\theta = \arcsin(2x)</math>. Writing <math>\sin(\theta) = 2x</math>, use implicit differentiation to show that</p> $\frac{d\theta}{dx} = \frac{A}{\sqrt{1 - Bx^2}}$ <p>where <math>A</math> and <math>B</math> are integers to be found.</p>	<p>Prove that for every real number <math>x \in \left[0, \frac{\pi}{2}\right]</math>, we have that <math>\sin(x) + \cos(x) \geq 1</math></p>	<p>Sketch, on the same axes <math>y = \frac{4}{x}</math> and <math>y = \frac{4}{x^2}</math>.</p>	<p>A particle of mass 2 kg is on a rough plane inclined at <math>30^\circ</math> to the horizontal. It is pulled up the plane by a force of 40 N. The coefficient of friction is <math>\mu = 0.5</math></p>
<p>Differentiate <math>y = x^2</math> from first principles.</p>	<p>A particle moves with velocity <math>v(t) = 3t^2 + 2te^t</math>. And the acceleration of the particle when <math>t = 1</math> second.</p>		<p>a) Draw a force diagram for this situation.</p> <p>b) Find the acceleration of the particle up the plane.</p>	
<p>A uniform beam <math>AB</math> rests on two supports <math>C</math> and <math>D</math> which are each 1 m from the ends <math>A</math> and <math>B</math> respectively. A box of mass 15 kg is placed on the beam at a distance of <math>x</math> metres from <math>A</math>. Given that the beam is on the point of tilting about <math>C</math> find the distance <math>x</math>.</p>	<p>Find the equation of the circle which has <math>A(3,6)</math> and <math>B(8, -4)</math> as the ends of a diameter.</p>	<p>The velocity time graph above shows the motion of a remote control car between <math>t = 0</math> and <math>t = 14</math> minutes.</p> <p>a) In what interval is the magnitude of the acceleration greatest?</p> <p>b) What is the acceleration in the interval above?</p> <p>c) Find the total distance travelled by the car?</p> <p>d) What is the displacement from the origin of the car after 14 seconds? Is this likely to be a realistic model?</p>	<p>Show that</p> $\int_1^4 x\sqrt{3x+1} dx = \frac{4}{135} (221\sqrt{13} - 18)$	

$$y = 3 \sin(2\theta)$$

$$= 6 \sin(\theta)\cos(\theta)$$

$$\Rightarrow y^2 = 36 \sin^2(\theta)\cos^2(\theta)$$

$$= 36x^2(1 - x^2)$$

Vertical component:  
 $u_V = 5 \sin(25^\circ)$   
 Horizontal component:  
 $u_H = 5 \cos(25^\circ)$

$|\mathbf{v}| = \sqrt{3^2 + 5^2}$   
 $= \sqrt{34}$   
 If  $\mathbf{v}$  makes an angle  $\theta$  with the vector  $\mathbf{i}$  then  $\theta = \arctan\left(\frac{5}{3}\right) \approx 59.03^\circ$ .  
 The unit vector parallel to  $\mathbf{v}$  is  
 $\frac{1}{\sqrt{34}}(3\mathbf{i} + 5\mathbf{j})$

$$gf(x) = \frac{3(3x + 6) - 1}{2(3x + 6)^2 - 1}$$

$$= \frac{9x + 17}{18x^2 + 72x + 71}$$

Hence,

$$\Rightarrow 9x + 17 = 0$$

$$\Rightarrow x = -\frac{17}{9} \approx -1.89$$

$$\left(\frac{6}{3}\right) 2^3 a^3 = 20$$

$$\Rightarrow 20 \times 8 \times a^3 = 20$$

$$\Rightarrow a^3 = \frac{1}{8}$$

$$\Rightarrow a = \frac{1}{2}$$

$$u_{10} = a + 9d = 35$$

$$u_{15} = a + 14d = 50$$

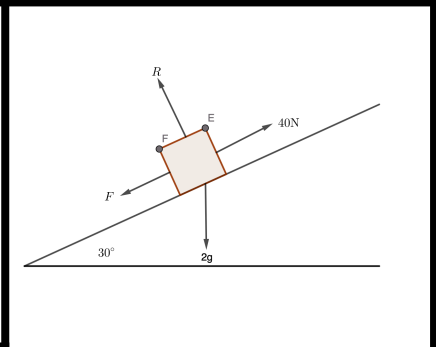
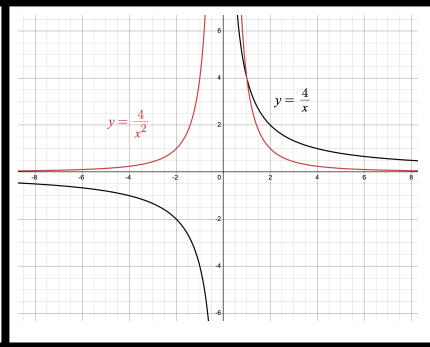
Solving these we obtain  
 $a = 8$  and  $d = 3$ . Hence,

$$S_{50} = \frac{50}{2} [2 \times 8 + 49 \times 3]$$

$$= 4050$$

$\theta = \arcsin(2x)$   
 $\Rightarrow \sin(\theta) = 2x$   
 $\cos(\theta) \frac{d\theta}{dx} = x$   
 $\Rightarrow \frac{d\theta}{dx} = \frac{2}{\cos(\theta)}$   
 So, differentiating,  
 $= \frac{2}{\sqrt{1 - \sin^2(\theta)}}$   
 $= \frac{2}{\sqrt{1 - 4x^2}}$

Assume, for a contradiction that there exists some  $x \in \left[0, \frac{\pi}{2}\right]$  where  
 $\sin(x) + \cos(x) < 1$ . Then we have  
 $0 < \sin(x) + \cos(x) < 1$ . Which means  
 $0 < (\sin(x) + \cos(x))^2 < 1$   
 $\Rightarrow 0 < \sin^2(x) + 2\sin(x)\cos(x) + \cos^2(x) < 1$   
 $\Rightarrow 0 < 1 + 2\sin(x)\cos(x) < 1$   
 So  $2\sin(x)\cos(x)$  which is a contradiction as both  $\sin(x)$  and  $\cos(x)$  are  $> 0$ . Hence the original result holds.



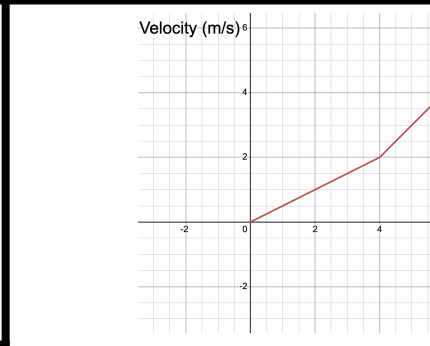
$$f'(x) = \lim_{h \rightarrow 0} \frac{(x+h)^2 - x^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - x^2}{h}$$

$$= \lim_{h \rightarrow 0} 2x + h$$

$$= 2x$$

$\frac{dv}{dt} = 6t + 2te^t + 2e^t$   
 When  $t = 1$ ,  
 $\frac{dv}{dt} = 6 + 4e$ . This is the acceleration of the particle.



Apply  $F = ma$  perpendicular to the plane:  
 $R - 2g \cos(30^\circ) = 0$   
 Apply  $F = ma$  parallel to the plane, up the plane:  
 $40 - F - 2g \sin(30^\circ) = 2a$   
 $\Rightarrow 40 - \frac{1}{2} \times 2g \cos(\theta) - 2g \sin(30^\circ) = 2a$   
 $\Rightarrow \frac{40 - \frac{1}{2} \times 2g \cos(\theta) - 2g \sin(30^\circ)}{2} = a$   
 so  $a \approx 10.8 \text{ ms}^{-2}$

As on the point of tilting about  $C$  the reaction at  $D$  is zero, so taking anticlockwise moments about  $C$ ,  
 $15(1 - x) = 3 \times 1$   
 $15 - 15x = 3$   
 $12 = 15x$   
 $x = \frac{12}{15}$

Midpoint =  $\left(\frac{3+8}{2}, \frac{6+(-4)}{2}\right) = \left(\frac{11}{2}, 1\right)$   
 radius =  $\sqrt{\left(8 - \frac{11}{2}\right)^2 + (3 - 1)^2}$   
 $= \sqrt{\frac{41}{4}}$   
 $= \frac{\sqrt{41}}{2}$   
 So circle is  $\left(x - \frac{11}{2}\right)^2 + (y - 1)^2 = \frac{41}{4}$

a) The acceleration is greatest when the line is steepest. This is when  $10 \leq t \leq 14$  as we are asked for the magnitude.  
 b) Acceleration is the gradient.  
 So  $a = \frac{5 - (-3)}{4} = \frac{8}{4} = 2 \text{ ms}^{-2}$   
 c) The area under the graph is

displacement.  
 So distance travelled =  $4 + 14 + 15 + 6.25 + 2.25 = 41.5 \text{ m}$ .  
 d) Displacement is signed difference so displacement is  $4 + 14 + 15 + 6.25 - 2.25 = 37 \text{ m}$ .  
 e) Instantaneous changes in acceleration are unlikely.

Let  $u = 3x + 1$ , then  $\frac{du}{dx} = 3$ , so  $dx = \frac{1}{3} du$ . Also  $x = \frac{u - 1}{3}$  and when  $x = 1$ ,  $u = 4$  and  $x = 4$ ,  $u = 7$ .  
 So, the integral becomes  
 $\int_4^7 \left(\frac{u-1}{3}\right) \sqrt{u} du$  from which the result follows.