AQA A-Level Further Mathematics Warmup - Paper 1 2024

Given that $2-3\mathrm{i}$ is a root of $p(x)=z^3-8z^2+29z-52$ find the other roots.	Given that $I_n=\int_0^1 x^n\mathrm{e}^x\ \mathrm{d}x$. Show that $I_n=\mathrm{e}-nI_{n-1}$ and find the exact value of I_3.	Find $\displaystyle\int \frac{1}{\sqrt{x^2-49}}\ \mathrm{d}x$	Prove by induction that, for all integers $n \geq 1$, $\displaystyle\sum_{r=1}^{n} r^3=\frac{n^2(n+1)^2}{4}$	Find the derivative of $y=\arcsin(3x-1)$
Sketch $\dfrac{x^2}{16}-9y^2=1$ showing it's asymptotes	Write down the key facts concerning a particle obeying SHM of the form $\ddot{x}=-\omega^2 x$	Sketch $r=2+3\sin(\theta)$	Find the area of the triangle with vertices $A(3,2,1)$, $B(6,3,4)$ and $C(8,2,3)$	Point P is reflected in the line $y=x$ and then rotated $90°$ anticlockwise to give the image point $(-2,3)$. What are the coordinates of P.
The polynomial $p(x)=x^3-7x+6$ has roots α, β and γ. Find the polynomial with roots $2\alpha-1$, $2\beta-1$ and $2\gamma-1$.	The determinant of $\begin{pmatrix} a & 2a \\ 4 & a \end{pmatrix}$ is -16. Find the possible values of a.	The graph of $y=\sqrt{x^2+1}$, $0 \leq x \leq 4$ is rotated 2π about the $x-$axis. Find the volume of the solid generated.	Find the equation of the straight line passing through $A(2,3,1)$ and $B(5,4,-3)$	Sketch $y=\dfrac{3x+4}{x-2}$
Find the mean value of the function $f(x)=2+3x^3$ over the interval $[2,6]$.	State Viète's formulae for the cubic equation $ax^3+bx^2+cx+d=0$ with roots α,β and γ.	Sketch $y=\sinh(x)$ and $y=\tanh(x)$	Find $I=\displaystyle\int \frac{3x^2+2x+52}{(x+2)(x^2+16)}\ \mathrm{d}x$	Find the Maclaurin series of $y=\mathrm{e}^x\cos(x)$ up to the term including x^4

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<td>Roots are $2-3\mathrm{i}$, $2+3\mathrm{i}$ and 4.</td>
<td>Let $u = x^n$ and $\frac{\mathrm{d}v}{\mathrm{d}x} = \mathrm{e}^x$ and use integration by parts to obtain the reduction formula.
$I_3 = 6 - 2\mathrm{e}$</td>
<td>$\operatorname{arcosh}\left(\frac{x}{6}\right) + C$</td>
<td>Proof</td>
<td>$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\sqrt{3}}{\sqrt{2x-3x^2}}$</td>
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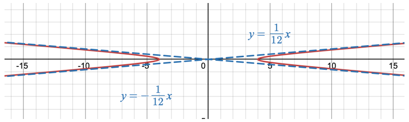
<td>
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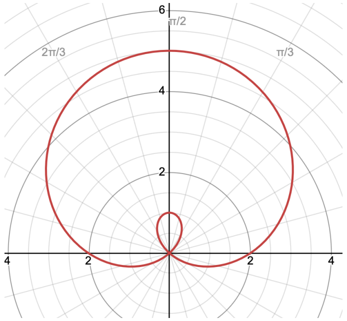
<td>General solution is $x = A\sin(\omega t) + B\cos(\omega t)$. Period, T of the particle is $T = \frac{2\pi}{\omega}$. if initially the object is on the central line then $x = a\sin(\omega t)$, if initially the object is the maximum displacement from the central line then the solution is $x = a\cos(\omega t)$. The velocity and displacement are related by $v^2 = \omega^2(a^2 - x^2)$</td>
<td>
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<td>$\frac{1}{2}\sqrt{110}$</td>
<td>Combined transformation matrix:
$\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} = \begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}$
So
$\begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}^{-1}\begin{pmatrix}-2\\ 3\end{pmatrix} = \begin{pmatrix}2\\ 3\end{pmatrix}$</td>
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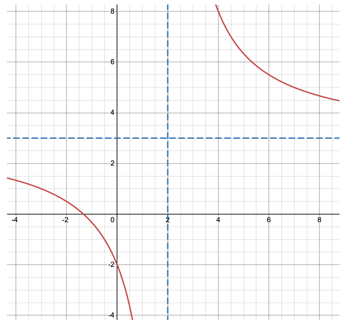
<td>Let $w = 2x - 1$ which implies that $x = \frac{w+1}{2}$ and substitute this into the original equation to give $p(w) = w^3 + 3w^2 - 25w + 21$</td>
<td>$a = 4$</td>
<td>Volume $= \pi\int_0^4 \left(\sqrt{xx^2+1}\right)^2\,\mathrm{d}x$
So, Volume $= \frac{76\pi}{3}$</td>
<td>$\mathbf{r} = \begin{pmatrix}2\\ 3\\ 1\end{pmatrix} + \lambda\begin{pmatrix}2\\ 1\\ -4\end{pmatrix}$</td>
<td></td>
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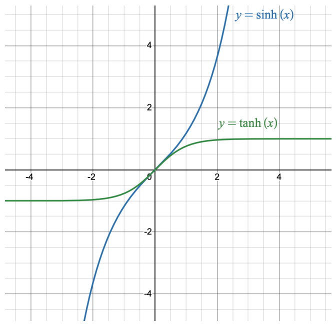
<td>242</td>
<td>$\alpha+\beta+\gamma = \frac{-b}{a}$
$\alpha\beta+\alpha\gamma+\beta\gamma = \frac{c}{a}$
$\alpha\beta\gamma = -\frac{d}{a}$</td>
<td>
</td>
<td>$\frac{3x^2+2x+52}{(x+2)(x^2+16)} = \frac{3}{x+2} + \frac{2}{x^2+16}$
So,
$I = 3\ln\lvert x+2\rvert + \frac{1}{2}\arctan\left(\frac{x}{4}\right)$</td>
<td>$y \approx 1 + x - \frac{x^3}{3} - \frac{x^4}{6}$</td>
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