AQA A-Level Further Mathematics Warmup - Paper 3 Mechanics and Discrete 2024

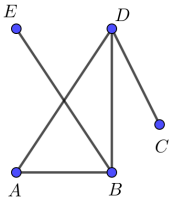
A car of mass 1200 kg is moving down a hill inclined at an angle θ where $\sin(\theta) = \frac{1}{30}$. The car is accelerating at 1.2ms^{-1} and the engine is working at a constant rate of 35 kW. Find the magnitude of the non-gravitational resistance to motion at the instant when the car is moving travelling at 5 ms^{-1}.

In Nottingham a new boutique patisserie opens. Each day the shop makes 500 items split between croissant and pain au raisin. They must make both items. Due to time constraints they can make no more than 400 croissants and no more than 300 pain au raisins. They make £1 profit on each croissant sold and £2 profit on each pain au raisin sold. The patisserie seeks to optimise their profit.

Set this up as a linear programming problem

A particle P of mass 2kg is moving at a speed of 4ms^{-1} collides with a particle Q of mass 3kg which is at rest. Given that after the collision P moves with speed 2ms^{-1}, find the speed of Q after the collision.

What is a planar graph? Show how the graph below can be shown to be planar.



A body moving on a horizontal circular path of radius r with a constant angular velocity has:

speed -
acceleration -
centripetal force -

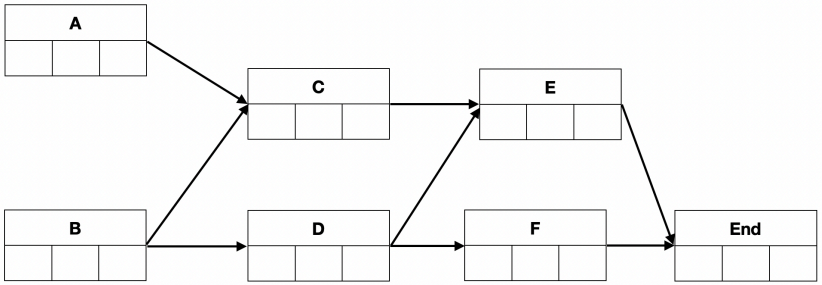
Let H be the group defined by the set $\{1, \mathrm{i}, -1, -\mathrm{i}\}$ under multiplication. Construct the Cayley table for the group H

Find the work done when a light elastic string of natural length 1.4 m and modulus of elasticity 60 N is stretched from a length of 1.5 m to 1.6m.

What is the rebound velocity for a particle with spreed $\mathbf{u} = a\mathbf{i} + b\mathbf{j}$ when it hits a wall parallel to $\mathbf{i}$ if the coefficient of restitution between the all and the particle is e.

For the tasks shown in the table to the right complete the activity network in the boxes below and identify the critical activities.

Task	Immediate Predecessors	Duration
A	-	3
B	-	5
C	A, B	4
D	B	3
E	C, D	5
F	D	4



What is Prim's Algorithm?

What is the centre of mass of the lamina with vertices $A(2,2)$, $B(5,6)$ and $C(7,3)$

In the topic of collisions how do you define the coefficient of restitution?

A particle P of mass 300g is attached to the lower end of a light inextensible string with the upper end fixed to th point A on the the ceiling. The string is at an angle of $60°$ to the downward vertical and the particle moves in a horizontal circle with centre 1.5m directly below A. Find the tension in the string and the speed of the particle.

What are the properties of a group $(G, *)$

John pulls a sledge along horizontal ground. The rope he is pulling has a tension of 60 N and the rope is at an angle of $30°$ to the horizontal. Find the work done pulling the sledge 10 m

How do you decide if a shape on an inclined plane will topple or slide?

State Lagrange's Theorem

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Let R be the non gravitational resistance to motion and T be the tractive force of the car. Using $P = Fv$, $T = 7000$ N. Applying $F = ma$ down the plane we have that
$T - R + 1200g\sin(\theta) = 1200 \times 1.2$
and so $R = 5952$.

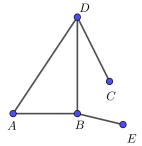
Let c be the number of croissants and p be the number of pain au raisins.
Then

Maximise $P : c + 2p$

$$\begin{aligned} c + p &= 500 \\ c &\leq 400 \\ p &\leq 300 \\ c &> 0 \\ p &> 0 \end{aligned}$$

Let the speed of Q after the collision be v.
Then,
$2 \times 4 + 3 \times 0 = 2 \times 2 + 3 \times v$
by conservation of momentum. So $v = \frac{4}{3}$ ms^{-1}

A graph is planar if it can be distorted so that its edges do not cross.



Speed: $v = r\omega$, constant along the tangent.
Acceleration: $a = r\omega^2 = \frac{v^2}{r}$ towards the centre.
Centripetal Force:
$F = mr\omega^2 = m\frac{v^2}{r}$

×	1	i	−1	−i
1	1	i	−1	−i
i	i	−1	−i	1
−1	−1	−i	1	i
−i	−i	1	i	−1

Work done
$= \frac{\lambda}{2l}(x_2^2 - x_1^2)$
So work done =
$= \frac{60}{2.8}(0.2^2 - 0.1^2) = \frac{9}{14}J$

$$\mathbf{v} = a\mathbf{i} - eb\mathbf{j}$$

Start by adding the durations into the middle of each boxes.
Then complete a forward pass to fill in the earliest start times. Then a backwards pass to fill in the latest finish times.

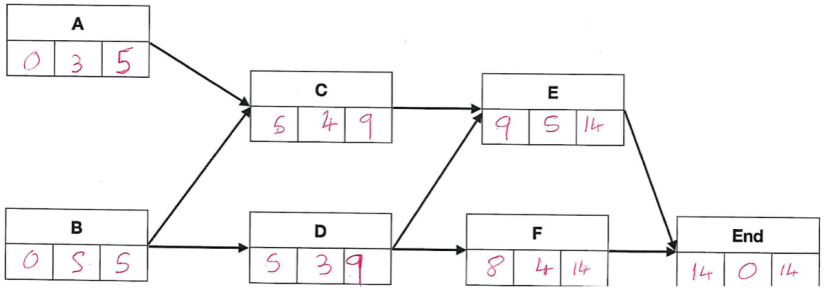
Task	Immediate Predecessors	Duration
A	-	3
B	-	5
C	A, B	4
D	B	3
E	C, D	5
F	D	4

1. Start with any node
2. Add the arc leading to the nearest node
3. Add the arc leading (from any of the nodes collected so far) to the nearest node and repeat.
4. Stop once all nodes are collected

$$\left(\frac{2+5+7}{3}, \frac{2+6+3}{3}\right)$$

$$= \left(\frac{14}{3}, \frac{11}{3}\right)$$

$$e = \frac{\text{Speed of separation}}{\text{Speed of approach}}$$
$$= \frac{v_2 - v_1}{u_1 - u_2}$$
where the velocities before impact are u_1 and u_2 and the velocities after the collision v_1 and v_2



Resolving vertically $T = \frac{0.3g}{\cos(60)} = 5.88N$
Applying $F = ma$, $T\sin(60) = \frac{mv^2}{r}$ so,
$v^2 = \frac{T\sin(60) \times 1.5\tan(60)}{0.3}$
$= \frac{441}{10}$
Hence $v \approx 6.64$ ms^{-1}

- G is a non-empty set.
- $*$ is a closed binary operation.
- $*$ is associative
- There is an identity element, e.
- Each $a \in G$ has as inverse $a^{-1} \in G$.

$$WD = 60 \times \cos(30) \times 10$$
$$\approx 520N$$

A shape on an incline will topple if the line of action of the centre of mass lies outside of the bottom edge or corner of the shape.
Suppose the plane is inclined at an angle θ to the horizontal and the coefficient of friction is μ, then the shape will slide before it topples if $\mu < (\tan(\theta))_{\text{topple}}$.

Lagrange's theorem states that the order of a subgroup of a finite group is a factor of the order of the group.