AQA A-Level Further Mathematics Warmup - Paper 3 Mechanics and Statistics 2024

A car of mass 1200 kg is moving down a hill inclined at an angle θ where $\sin(\theta) = \frac{1}{30}$. The car is accelerating at 1.2ms^{-1} and the engine is working at a constant rate of 35 kW. Find the magnitude of the non-gravitational resistance to motion at the instant when the car is moving travelling at 5 ms^{-1}.	A CRV X, has probability density function given by $f(x) = \begin{cases} 3x^a; & 0 \leq x \leq 1 \\ 0; & \text{otherwise} \end{cases}$ Find the constant a and the median value M, of X.	A particle P of mass 2kg is moving at a speed of 4ms^{-1} collides with a particle Q of mass 3kg which is at rest. Given that after the collision P moves with speed 2ms^{-1}, find the speed of Q after the collision.	Define Type I and Type II errors	A body moving on a horizontal circular path of radius r with a constant angular velocity has: speed - acceleration - centripetal force -
How do you calculate the expectation of a continuous random variable?	Find the work done when a light elastic string of natural length 1.4 m and modulus of elasticity 60 N is stretched from a length of 1.5 m to 1.6m.	What is the rebound velocity for a particle with spreed $\mathbf{u} = a\mathbf{i} + b\mathbf{j}$ when it hits a wall parallel to $\mathbf{i}$ if the coefficient of restitution between the all and the particle is e.	When would you use a $t-$test? And what is the formula for the test statistic?	Random events occur at a rate of 3 per minute. a) Write the probability density function $f(t)$ and the cumulative density function $F(t)$ for the random variable T, the waiting time in minute between events. b) What is the mean and variance of T.
Derive the mean for a discrete uniform distribution $X \sim U(n)$. State the formula for the $Var(X)$	What is the centre of mass of the lamina with vertices $A(2,2)$, $B(5,6)$ and $C(7,3)$	In the topic of collisions how do you define the coefficient of restitution?	What is the formula for Yate's correction?	A Geiger counter detects radioactive decays at a mean rate of 25 per minute. Find the probability that in a given, randomly chosen minute, there are i) 22 decays ii) More than 27 decays
A particle P of mass 300g is attached to the lower end of a light inextensible string with the upper end fixed to th point A on the the ceiling. The string is at an angle of $60°$ to the downward vertical and the particle moves in a horizontal circle with centre 1.5m directly below A. Find the tension in the string and the speed of the particle.	If X and Y are Poisson random variables then what is the distribution of $X + Y$?	John pulls a sledge along horizontal ground. The rope he is pulling has a tension of 60 N and the rope is at an angle of $30°$ to the horizontal. Find the work done pulling the sledge 10 m	How do you decide if a shape on an inclined plane will topple or slide?	Prove that the exponential distribution $f(x) = \lambda e^{-\lambda x}$, with $x \geq 0$ has a mean of $\frac{1}{\lambda}$.

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Let R be the non gravitational resistance to motion and T be the tractive force of the car. Using $P = Fv$, $T = 7000$ N. Applying $F = ma$ down the plane we have that
$T - R + 1200g\sin(\theta) = 1200 \times 1.2$
and so $R = 5952$.

$\int_0^1 3x^a \, dx = 1 \quad \Rightarrow \quad a = 2$

$\int_0^M 3x^2 \, dx = \frac{1}{2}$

$\Rightarrow \quad M^3 = \frac{1}{2}$

$\Rightarrow \quad M \approx 0.7937$

Let the speed of Q after the collision be v.
Then,
$2 \times 4 + 3 \times 0 = 2 \times 2 + 3 \times v$
by conservation of momentum. So $v = \frac{4}{3}$ ms^{-1}

A type I error is falsely rejecting H_0.

A type II error is not rejecting H_0 when it is false.

Speed: $v = r\omega$, constant along the tangent.

Acceleration: $a = r\omega^2 = \frac{v^2}{r}$ towards the centre.

Centripetal Force:
$F = mr\omega^2 = m\frac{v^2}{r}$

$$E[x] = \int_{-\infty}^{\infty} xf(x) \, dx$$

Work done
$= \frac{\lambda}{2l}(x_2^2 - x_1^2)$
So work done =
$= \frac{60}{2.8}(0.2^2 - 0.1^2) = \frac{9}{14}J$

$$\mathbf{v} = a\mathbf{i} - eb\mathbf{j}$$

Suppose a sample of size n is taken from a distribution. We use a $t-$test if the population variance is unknown and we only know the sample variance s^2. In this case the test statistic is $T = \frac{\bar{x} - \mu_0}{\frac{S}{\sqrt{n}}}$ and it follows a $t-$distribution with $n - 1$ degrees of freedom.

Exponential distribution
$f(t) = 3e^{-3t}, t \geq 0$
$F(t) = 1 - e^{-3t}, t \geq 0$
Mean: $\frac{1}{\mu} = \frac{1}{3}$
Variance: $\frac{1}{\mu^2} = \frac{1}{9}$

$E[x] = \sum_{r=1}^{n} r \times \frac{1}{n}$
$= \frac{1}{n}\frac{r(n+1)}{2}$
$= \frac{n+1}{2}$
$\text{Var}(x) = \frac{n^2 - 1}{12}$

$$\left(\frac{2+5+7}{3}, \frac{2+6+3}{3}\right)$$
$$= \left(\frac{14}{3}, \frac{11}{3}\right)$$

$e = \frac{\text{Speed of separation}}{\text{Speed of approach}}$
$= \frac{v_2 - v_1}{u_1 - u_2}$
where the velocities before impact are u_1 and u_2 and the velocities after the collision v_1 and v_2

$$\chi^2_{\text{Yates}} = \sum \frac{\left(|O_i - E_i| - 0.5\right)^2}{E_i}$$

Let $X \sim \text{Po}(25)$. Then
$P(X = x) = e^{-25}\frac{25^x}{x!}$.
$P(X = 22) = 0.0702$
$\text{P}(X > 27) = 1 - \text{P}(X \leq 27)$
$= 1 - 0.7001$
$= 0.2998$

Resolving vertically $T = \frac{0.3g}{\cos(60)} = 5.88N$
Applying $F = ma$, $T\sin(60) = \frac{mv^2}{r}$ so,
$v^2 = \frac{T\sin(60) \times 1.5\tan(60)}{0.3}$
$= \frac{441}{10}$
Hence $v \approx 6.64$ ms^{-1}

$$X + Y \sim \text{Po}(\lambda_1 + \lambda_2)$$

$$WD = 60 \times \cos(30) \times 10$$
$$\approx 520N$$

A shape on an incline will topple if the line of action of the centre of mass lies outside of the bottom edge or corner of the shape.
Suppose the plane is inclined at an angle θ to the horizontal and the coefficient of friction is μ, then the shape will slide before it topples if $\mu < (\tan(\theta))_{\text{topple}}$.

$\text{E}[X] = \int_{-\infty}^{\infty} xf(x) \, dx$
$= \int_0^{\infty} x\lambda e^{-\lambda x}$
$= \left[-xe^{-\lambda x}\right]_0^{\infty} - \left[\frac{1}{\lambda}e^{-\lambda x}\right]_0^{\infty}$
$= \frac{1}{\lambda}$
using integration by parts..