AQA A-Level Further Mathematics Warmup - Paper 3 Statistics and Discrete 2024

A CRV X, has probability density function given by

$$f(x) = \begin{cases} 3x^a; & 0 \le x \le 1 \\ 0; & \text{otherwise} \end{cases}$$

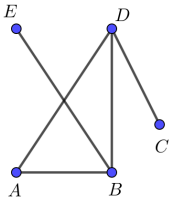
Find the constant a and the median value M, of X.

In Nottingham a new boutique patisserie opens. Each day the shop makes 500 items split between croissant and pain au raisin. They must make both items. Due to time constraints they can make no more than 400 croissants and no more than 300 pain au raisins. They make £1 profit on each croissant sold and £2 profit on each pain au raisin sold. The patisserie seeks to optimise their profit.

Set this up as a linear programming problem

Define Type I and Type II errors

What is a planar graph? Show how the graph below can be shown to be planar.



When would you use a $t-$ test? And what is the formula for the test statistic?

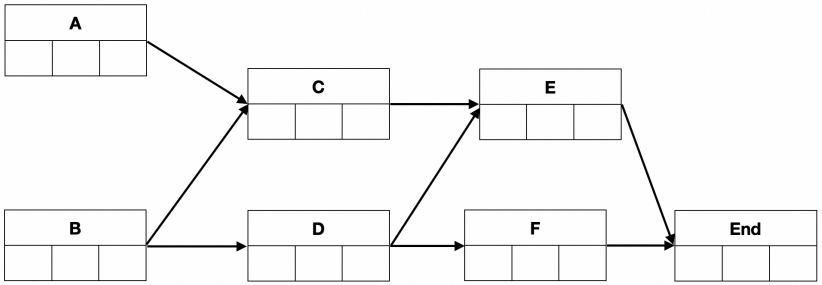
Let H be the group defined by the set $\{1, \mathrm{i}, -1, -\mathrm{i}\}$ under multiplication. Construct the Cayley table for the group H

Random events occur at a rate of 3 per minute.
a) Write the probability density function $f(t)$ and the cumulative density function $F(t)$ for the random variable T, the waiting time in minute between events.
b) What is the mean and variance of T.

How do you calculate the expectation of a continuous random variable?

For the tasks shown in the table to the right complete the activity network in the boxes below and identify the critical activities.

Task	Immediate Predecessors	Duration
A	-	3
B	-	5
C	A, B	4
D	B	3
E	C, D	5
F	D	4



What is Prim's Algorithm?

If X and Y are Poisson random variables then what is the distribution of $X + Y$?

What is the formula for Yate's correction?

Derive the mean for a discrete uniform distribution $X \sim U(n)$. State the formula for the $Var(X)$

What are the properties of a group $(G, *)$

Prove that the exponential distribution $f(x) = \lambda \mathrm{e}^{-\lambda x}$, with $x \ge 0$ has a mean of $\frac{1}{\lambda}$.

A Geiger counter detects radioactive decays at a mean rate of 25 per minute. Find the probability that in a given, randomly chosen minute, there are
i) 22 decays
ii) More than 27 decays

State Lagrange's Theorem

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$$\int_0^1 3x^a \, \mathrm{d}x = 1 \quad \Rightarrow \quad a = 2$$
$$\int_0^M 3x^2 \, \mathrm{d}x = \frac{1}{2}$$
$$\Rightarrow \quad M^3 = \frac{1}{2}$$
$$\Rightarrow \quad M \approx 0.7937$$

Let c be the number of croissants and p be the number of pain au raisins.
Then

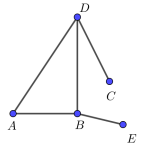
Maximise $P : c + 2p$

$$\begin{aligned} c + p &= 500 \\ c &\leq 400 \\ p &\leq 300 \\ c &> 0 \\ p &> 0 \end{aligned}$$

A type I error is falsely rejecting H_0.

A type II error is not rejecting H_0 when it is false.

A graph is planar if it can be distorted so that its edges do not cross.



Suppose a sample of size n is taken from a distribution. We use a $t-$test if the population variance is unknown and we only know the sample variance s^2. In this case the test statistic is $T = \dfrac{\bar{x} - \mu_0}{\dfrac{S}{\sqrt{n}}}$ and it follows a $t-$distribution with $n - 1$ degrees of freedom.

×	1	i	−1	−i
1	1	i	−1	−i
i	i	−1	−i	1
−1	−1	−i	1	i
−i	−i	1	i	−1

Exponential distribution
$f(t) = 3\mathrm{e}^{-3t}, t \geq 0$
$F(t) = 1 - \mathrm{e}^{-3t}, t \geq 0$
Mean: $\dfrac{1}{\mu} = \dfrac{1}{3}$
Variance: $\dfrac{1}{\mu^2} = \dfrac{1}{9}$

$$E[x] = \int_{-\infty}^{\infty} x f(x) \, \mathrm{d}x$$

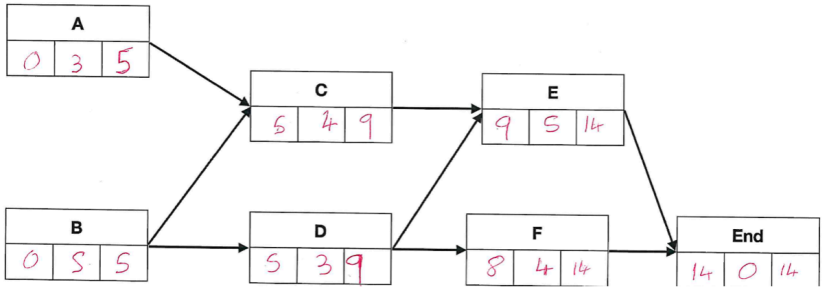
Start by adding the durations into the middle of each boxes.
Then complete a forward pass to fill in the earliest start times. Then a backwards pass to fill in the latest finish times.

Task	Immediate Predecessors	Duration
A	-	3
B	-	5
C	A, B	4
D	B	3
E	C, D	5
F	D	4

1. Start with any node
2. Add the arc leading to the nearest node
3. Add the arc leading (from any of the nodes collected so far) to the nearest node and repeat.
4. Stop once all nodes are collected

$$X + Y \sim \mathrm{Po}(\lambda_1 + \lambda_2)$$

$$\chi^2_{\text{Yates}} = \sum \frac{\left(|O_i - E_i| - 0.5\right)^2}{E_i}$$



$$E[x] = \sum_{r=1}^{n} r \times \frac{1}{n}$$
$$= \frac{1}{n} \frac{r(n+1)}{2}$$
$$= \frac{n+1}{2}$$
$$\mathrm{Var}(x) = \frac{n^2 - 1}{12}$$

- G is a non-empty set.
- $*$ is a closed binary operation.
- $*$ is associative
- There is an identity element, e.
- Each $a \in G$ has as inverse $a^{-1} \in G$.

$$\begin{aligned} \mathrm{E}[X] &= \int_{-\infty}^{\infty} x f(x) \, \mathrm{d}x \\ &= \int_0^{\infty} x \lambda \mathrm{e}^{-\lambda x} \\ &= \left[-x\mathrm{e}^{-\lambda x}\right]_0^{\infty} - \left[\frac{1}{\lambda}\mathrm{e}^{-\lambda x}\right]_0^{\infty} \\ &= \frac{1}{\lambda} \end{aligned}$$

using integration by parts..

Let $X \sim \mathrm{Po}(25)$. Then
$P(X = x) = \mathrm{e}^{-25}\dfrac{25^x}{x!}$.
$P(X = 22) = 0.0702$
$\begin{aligned} \mathrm{P}(X > 27) &= 1 - \mathrm{P}(X \leq 27) \\ &= 1 - 0.7001 \\ &= 0.2998 \end{aligned}$

Lagrange's theorem states that the order of a subgroup of a finite group is a factor of the order of the group.